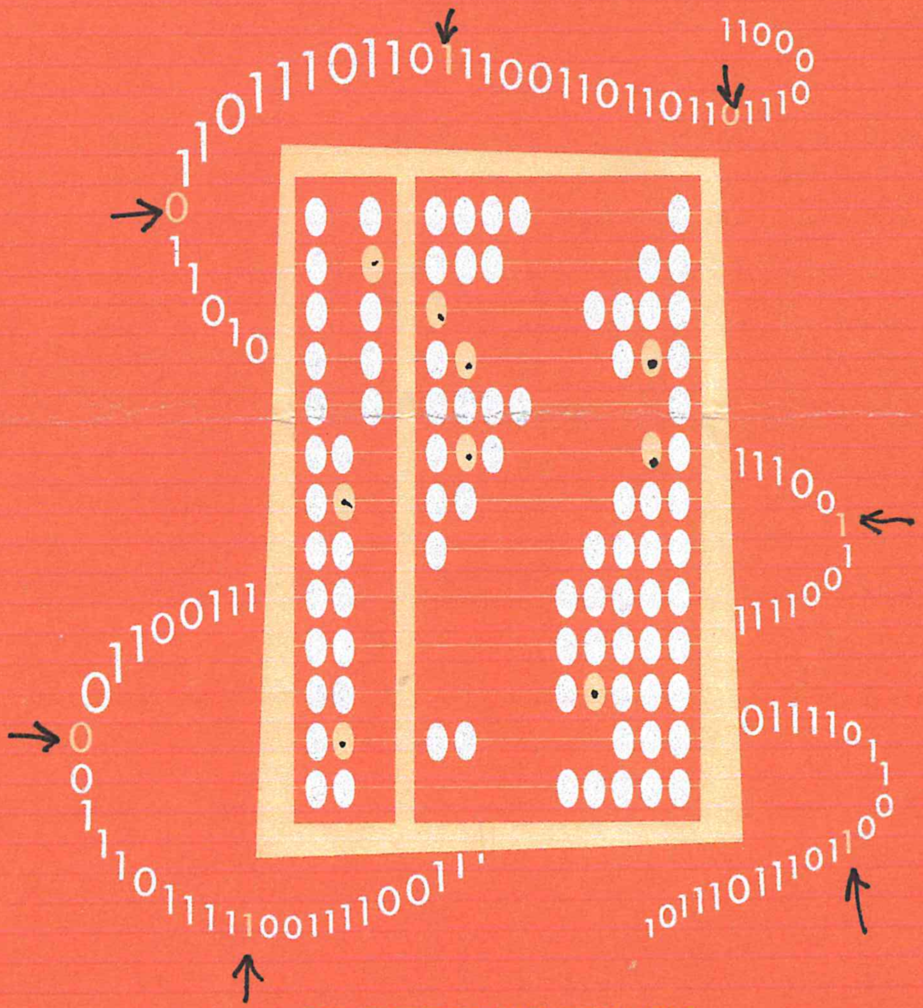
UNDERSTANDING MODERN MATHEMATICS

BASES AND NUMERALS



AN INTRODUCTION TO

NUMERATION

A MACMILLAN PROGRAMMED TEXT

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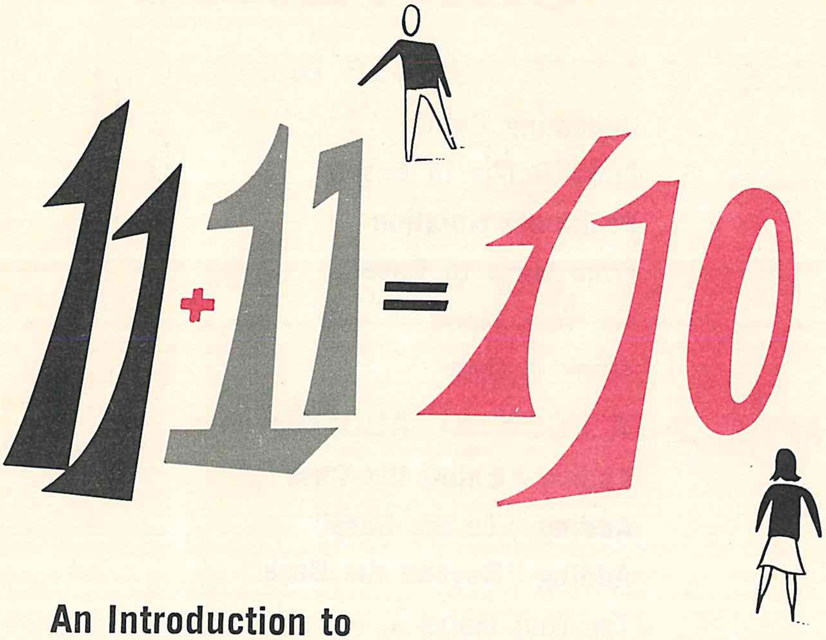

An Introduction to

NUMERATION

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About This Book

■ This is not an ordinary textbook. Nor is it even an ordinary workbook. It is called a *program.* It makes use of a teaching method called *programmed instruction* to help you learn better. In this program you'll see a series of numbered sections called *frames* and in each frame you will be asked a question. You'll learn by answering the questions.

There are two important things you should know about programmed instruction. The first is that a program is *not* a test. You won't be graded on how many questions you answer correctly. When you finish answering a question you are allowed to lift the flap of the Flexitab© binder to look at the correct answer. (But be sure not to look at the printed answer until you have finished giving your own answer.)

The second important thing is that you should work at your own speed. If you take your time and answer carefully you'll learn better than if you "race" through the program.

In this program the questions fall into two types: fill-in and multiple-choice. You answer fill-in questions by writing your answer in the blank. You answer multiple-choice questions by *circling* your choice. Then you lift the flap and look at the answer column to find out whether you answered correctly. If you did, you should go on to the next frame. If you didn't, you should cross out the wrong answer and write the correct one. Here are two examples to get you started:

1. The first president of the U. S. was George ____________________.

Washington

2. The second president of the U. S. was John ——?——.

   a. Hamilton   b. Adams   c. Roosevelt

b. Adams

Did you answer correctly? If not, go back and write the correct answers.

Now you can begin the program. You start on page 1 and use the right-hand pages. The left-hand pages will be upside down and you needn't even look at them. When you get to the end of the book, turn the program upside down and continue through the rest of the frames. If you have any questions about the program, be sure to ask your teacher.

1 Other Number Bases

STRANGE NUMBER FACTS

■ Suppose one of your classmates came up to you and said, "**4 + 3 = 10.**" You would probably figure that he had forgotten his arithmetic facts. It would be hard for you to understand how a person could make such a statement as "**4 + 3 = 10.**"

In this book you are going to see that sometimes 4 + 3 *does* equal 10. You will also see that sometimes **4 + 3 = 11** and sometimes **4 + 3 = 12.** The mystery behind all these statements will be unfolded for you in this book. You will soon see why a statement such as **4 + 3 = 12** can be true even though it sounds ridiculous to you at the present time.

The subject you are going to study involves much more than "unusual" arithmetic facts. You will be taking a brand new look at much of the arithmetic you already know, but before long you will find yourself discovering new arithmetic facts, methods, and patterns. For what might be the first time in your life, you will be *understanding* mathematics rather than simply memorizing facts and methods.

The world of mathematics plays a role in almost every great advancement in today's modern space age. You are now going to take your first step into an interesting and important part of this world. You are going to study about **number bases.**

Collecting Coins

1. One day, Pete's father gave him 2 quarters, 3 nickels, and 1 penny. Altogether, Pete's father gave him —?— cents.

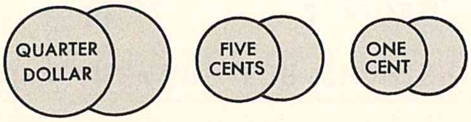
   a. 231 b. 66 c. 6

   b. 66

2. Pete's sister, Monica, had 1 quarter, 2 nickels, and 4 pennies. Altogether, Monica had ______ cents.

   39

3. Here is a picture of some quarters, nickels, and pennies:



   The value of these coins is ______ cents.

   62

4. The value of a silver dollar, a dime, and a penny would be 111 cents. And the value of 2 silver dollars, 3 dimes, and a penny would be ______ cents.

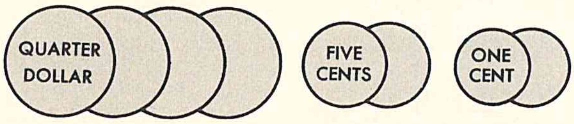
   231

5. A silver dollar, 4 dimes, and 4 pennies would have a value of ______ cents.

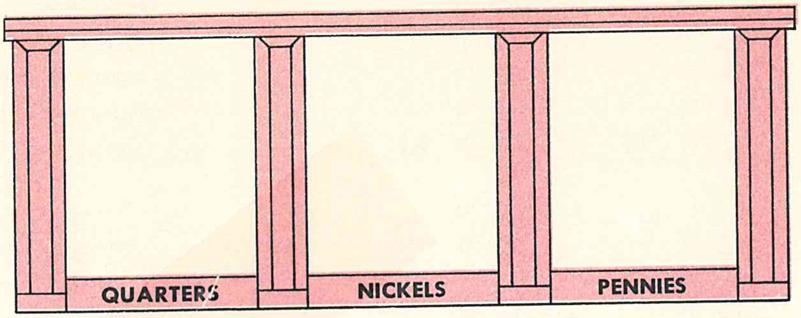
   144

6. If Monica had the coins shown below, she would have ______ cents.

   112



7. Pete saved his money in a bank that looked like this:



He put only quarters in the quarter holder, only nickels in the nickel holder, and only pennies in the penny holder. He couldn't put dimes in any of the holders. In order to save a dime, he would

change it to ______ nickels. 2

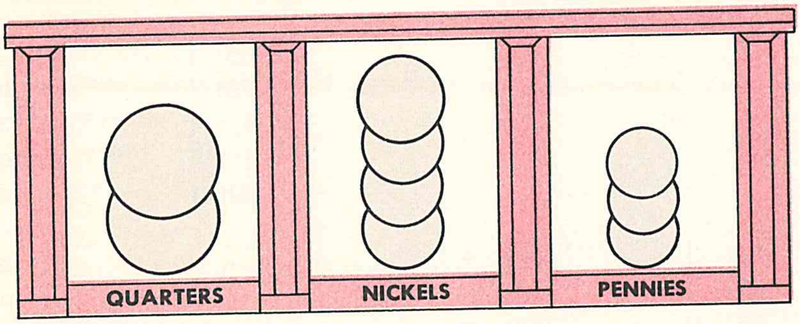
8. If he wanted to save a fifty-cent coin, he would change it to

______ quarters. 2

9. And if Pete's father had given him a dollar bill, he could have

saved it in his bank by changing it to ______ quarters. 4

10. If Pete had 2 quarters, 4 nickels, and 3 pennies, the coins in his bank would look like this:

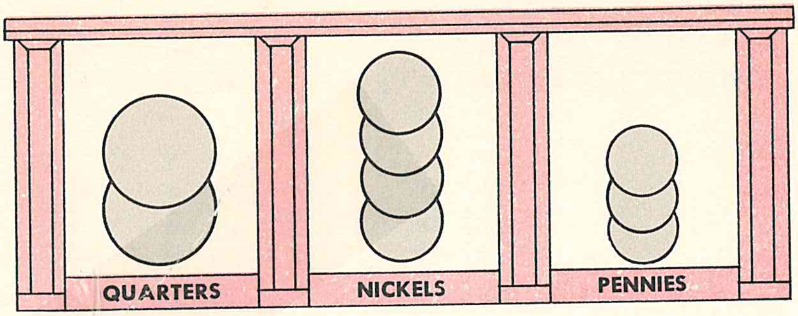


In the quarters holder, there are ______ quarters. In the nickels 2

holder, there are ______ nickels. And in the pennies holder, there 4

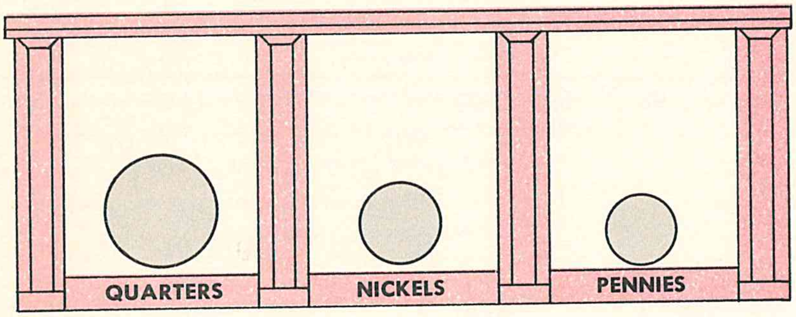
are ______ pennies. 3

11. The picture below represents ______ quarters, ______ nickels, and ______ pennies. 2, 4 3

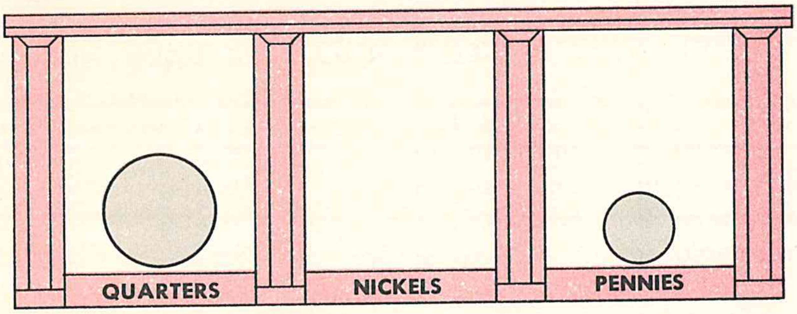


12. The value of the money in the last frame is ______ cents. 73

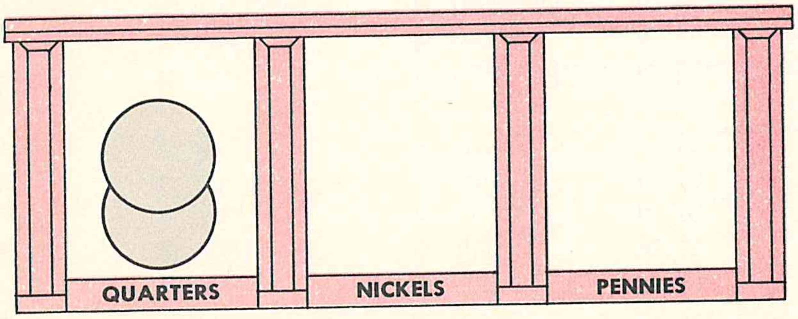
13. The value of the money shown below is ______ cents. 31



14. The picture below shows ______ cents. 26



15. The value of the money in the bank shown below is ______ cents. 50



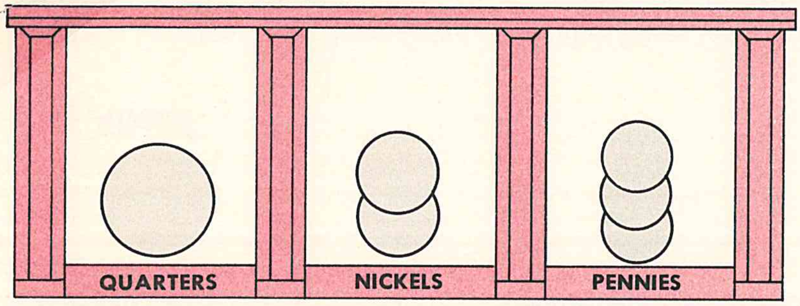
16. When Pete described his fortune to Monica, he used numerals arranged in the same order as the holders in his bank. For example, **2 3 1** (read "two, three, one," *not* "two hundred thirty-one") meant

2 quarters, 3 nickels, and 1 penny. Altogether, Pete had ______ cents. 66

17. If Pete were to say **2 4 3,** he would mean ______ quarter(s), ______ 2, 4

nickel(s), and ______ penny(ies). 3

18. One day Pete's bank looked like this:

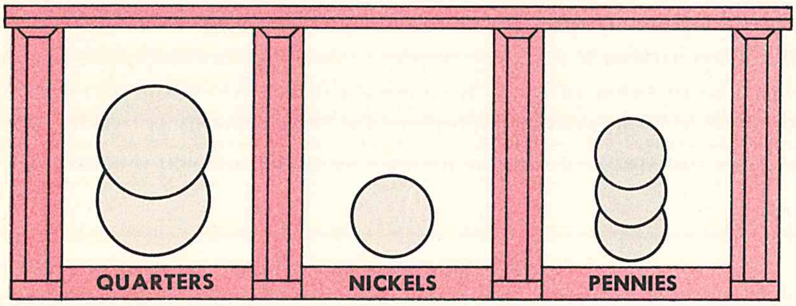


Pete would say **1 2 3** to Monica to mean that he had ______ 1

quarter(s), ______ nickel(s), and ______ penny(ies). 2, 3

19. Pete would describe the bank below by saying —?—.

a. 3 1 2 **b.** 1 2 3 **c.** 2 1 3 — **c.** 2 1 3



20. If Pete had only 2 pennies in his bank, he'd say **2** to Monica. And

if he had only 3 pennies in the bank, he'd say ______. 3

21. If he had a nickel and a penny, he would say —?—.

a. 5 1 **b.** 1 1 — **b.** 1 1

22. In Pete's system, 1 nickel and 2 pennies would be written as ________. 1 2

23. Pete would say **1 0** (read "one, zero") to represent ______ nickel(s) and ______ penny(ies). 1 0

24. If Pete had a quarter, a nickel, and a penny, he'd say **1 1 1.** If he had only a quarter and a nickel, he'd say **1 1 0.** If he wanted to show that he had 2 quarters and 1 nickel, he'd have to say ——?——.

a. 2 1 0 **b.** 1 0 2 **c.** 21 — **a.** 2 1 0

25. The meaning of **2 0 1** in Pete's system is ______ quarter(s), ______ nickel(s), and 1 penny. 2 0

26. The meaning of **3 0 0** in Pete's system is ______ quarters, ______ nickels and ______ penny(ies). 3, 0 0

27. In Pete's system, 3 nickels would be written as ——?——.

a. 3 **b.** 3 0 — **b.** 3 0

8. In Pete's system, 2 nickels would be written as ________. 2 0

9. In Pete's system, 4 nickels would be written as ——?——.

a. 4 **b.** 4 0 **c.** 4 0 0 — **b.** 4 0

0. In Pete's system, 2 quarters and 2 pennies would be written as ________. 2 0 2

31. To mean 0 quarters, 2 nickels, and 3 pennies, Pete wouldn't say **0 2 3.** He'd simply say ________. 2 3

32. Pete would say __________ to mean 3 quarters and 1 penny. 3 0 1

33. Represent each of the following in Pete's system:

1. 2 quarters and 2 pennies __________ 2 0 2
2. 4 quarters and 1 nickel __________ 4 1 0
3. 1 nickel __________ 1 0
4. 3 quarters __________ 3 0 0
5. 2 quarters and 1 nickel __________ 2 1 0

34. Here are some numerals that Pete used to describe his savings to Monica. Find the number of quarters, nickels, and pennies he had in each case.

1. **3 0 2** ______ quarters 3
   ______ nickels 0
   ______ pennies 2
2. **1 3 4** ______ quarters 1
   ______ nickels 3
   ______ pennies 4
3. **4** ______ quarters 0
   ______ nickels 0
   ______ pennies 4
4. **4 0 0** ______ quarters 4
   ______ nickels 0
   ______ pennies 0

5.	40	_____ quarters	0
		_____ nickels	4
		_____ pennies	0

35. In each case below, tell how many cents Pete had.

1.	3 0 0	_____ cents	75
2.	1 0 3	_____ cents	28
3.	1 0 0	_____ cents	25
4.	2 1 0	_____ cents	55
5.	4 4 4	_____ cents	124
6.	1 0	_____ cents	5
7.	2 1	_____ cents	11
8.	1 4	_____ cents	9
9.	3 0 2	_____ cents	77
10.	4	_____ cents	4

36. Now, let's suppose Pete received 11 cents from his dad. He could put 11 cents into his bank in several different ways. For example, he could put in _____ nickel(s) and 1 penny. 2

37. He could also put in 1 nickel and _____ pennies. 6

38. A third way would be to put in _____ pennies. 11

39. If Pete uses 2 nickels and 1 penny, he only puts in _____ coins. 3

40. With 1 nickel and 6 pennies, he puts ______ coins into his bank. 7

41. If he uses only pennies, then he puts ______ coins into the bank. 11

42. Since there are three ways of changing 11 cents, let's agree to use the one that requires the *least number* of coins. That is, if Pete wants to save 11 cents, we'll have him put ______ nickel(s) and ______ penny(ies) into his bank. 2 1

43. 17 cents may be put into Pete's bank in any one of the following combinations:

1. 3 nickels and ______ pennies 2
2. 2 nickels and ______ pennies 7
3. 1 nickel and ______ pennies 12
4. 0 nickels and ______ pennies 17

44. The way to use the least number of coins in putting 17 cents into Pete's bank is to use ______ nickels and ______ pennies. 3, 2

45. We write 18 cents in Pete's system as ——?——.

a. 2 8 **b.** 3 3 **b.** 3 3

46. In Pete's system, 22 cents is represented as ——?——.

a. 4 2 **b.** 3 7 **a.** 4 2

47. In Pete's system, 19 cents is written as ________. 3 4

48. In Pete's system, 24 cents is ________. 4 4

49. Using the least number of coins, Pete can put 25 cents into his bank by using ______ quarter(s), ______ nickel(s), and ______ penny(ies). 1, 0, 0

50. In Pete's system, 25 cents can be expressed as —?—.

a. 1 0 0 **b.** 5 0 **c.** 2 5 **a.** 1 0 0

51. In Pete's system, 27 cents is written as —?—.

a. 1 0 2 **b.** 2 0 1 **c.** 5 2 **a.** 1 0 2

52. And in Pete's system, 30 cents is written as ________. 1 1 0

53. Write in Pete's system each of the following amounts of money. Use the smallest number of coins.

1.	78 cents	________	3 0 3
2.	12 cents	________	2 2
3.	23 cents	________	4 3
4.	100 cents	________	4 0 0
5.	50 cents	________	2 0 0
6.	114 cents	________	4 2 4
7.	80 cents	________	3 1 0
8.	42 cents	________	1 3 2
9.	8 cents	________	1 3
10.	28 cents	________	1 0 3

From Banks to Bases

54. Instead of using the words *quarters, nickels,* and *pennies,* as Pete did, we're going to make a new system, in which we'll use the word **one** instead of Pete's *penny.* That is, we'll say 4 —?— instead of 4 pennies.

 a. ones **b.** cents

a. ones

55. Just as 5 of Pete's pennies make a nickel in our system, 5 *ones* will make a **five.** That is, instead of the word *nickel,* we'll use the word —?—.

 a. one **b.** five

b. five

56. Where Pete would say "2 nickels and 3 pennies," we're going to say "2 ________ and 3 ________."

fives, ones

57. And 3 fives and 2 ones would be the same as ____ ____________ and ____ ____________ in Pete's system.

3 nickels
2 pennies

58. When we write **1 4** in our new system, we mean 1 five and 4 ones. When we write **4 3,** we mean ______ five(s) and ______ one(s).

4, 3

59. We're really using Pete's system, except that we're using fives and ones instead of the ____________ and ____________ that Pete used.

nickels, pennies

60. Pete wrote **1 3** to mean eight cents. When we write **1 3,** we'll mean eight, because **1 3** means ______ five(s) and ______ one(s).

1, 3

61. Ordinarily, when you see a numeral such as 23, you read it as "twenty-three" and you mean 2 *tens* and 3 *ones.*

 The numeral 32 means 3 ________ and 2 ________.

tens, ones

62. We say numerals such as 23 and 32 are written in **base ten.** It is called the base ten system because the **2** in the numeral **23** means 2 ——?——.

a. ones **b.** fives **c.** tens

c. tens

63. When Pete wrote **1 3** to mean a nickel and 3 pennies, he was using a base system other than base ten. He was using base ——?——.

a. one **b.** five **c.** six

b. five

64. If you write **2 3** in base five, you mean _____ five(s) and _____ one(s).

2, 3

65. So that you will know that **2 3** is in base five when you see it, we'll write the name of the base in letters after the numeral. For example, **2 3** in base five will be written as 23_{five}.

The numeral 14_{six} is written in base _______.

six

66. The numeral 25_{seven} is written in base _______.

seven

67. And the numeral 231_{nine} is in base _______.

nine

68. Just as you were able to understand Pete's way of describing his savings, you will be able to understand numerals written in other bases.

For example, 23_{five} corresponds to **2 3** in Pete's coin system.

That is, 23_{five} means _____ five(s) and _____ one(s).

2, 3

69. If we translate 23_{five} to base ten, then, we get ——?——.

a. 13_{ten} **b.** 23_{ten} **c.** 25_{ten}

a. 13_{ten}

70. When we write 23_{six}, we mean _____ sixes and _____ ones.

2, 3

71. So 23_{six} means ——?—— in base ten. c. 15_{ten}

a. 23_{ten} b. 18_{ten} c. 15_{ten}

72. The numeral 23_{four} means 2 fours and ______ one(s). 3

73. If we want to transform 23_{four} to base ten, we add 2 fours to 3 ones. This sum, written in base ten, is 11_{ten}.

If we want to transform 12_{four} to base ten, we add ______ 1

four(s) to ______ one(s). This sum in base ten is ____ $_{ten}$. 2, 6

74. The value in base ten of 32_{four} is ______ $_{ten}$. 14

75. The numeral 26_{seven} means ______ seven(s) and ______ one(s). 2, 6

So 26_{seven} is the same as ______ $_{ten}$. 20

76. The numeral 10_{seven} is ____ $_{ten}$. 7

77. Transform each of the following numerals to base ten:

1. $10_{three} =$ ____ ten 3
2. $10_{nine} =$ ____ ten 9
3. $10_{eight} =$ ____ ten 8
4. $10_{two} =$ ____ ten 2
5. $10_{six} =$ ____ ten 6

78. Transform each of the following numerals to base ten:

1. $20_{\text{three}} = \_\_\_\_\_\__{\text{ten}}$ 6
2. $30_{\text{nine}} = \_\_\_\_\_\__{\text{ten}}$ 27
3. $40_{\text{seven}} = \_\_\_\_\_\__{\text{ten}}$ 28
4. $50_{\text{six}} = \_\_\_\_\_\__{\text{ten}}$ 30
5. $70_{\text{nine}} = \_\_\_\_\_\__{\text{ten}}$ 63

79. Change each of the following numerals to base ten:

1. $21_{\text{three}} = \_\_\_\_\_\__{\text{ten}}$ 7
2. $14_{\text{six}} = \_\_\_\_\_\__{\text{ten}}$ 10
3. $32_{\text{nine}} = \_\_\_\_\_\__{\text{ten}}$ 29
4. $55_{\text{seven}} = \_\_\_\_\_\__{\text{ten}}$ 40
5. $17_{\text{eight}} = \_\_\_\_\_\__{\text{ten}}$ 15

Positional Notation

80. In base ten, if you see the numeral 2_{ten}, you know that the 2 means 2 ones. And if you see 20_{ten}, you know that the 2 means 2 tens. In other words, the *position* of the 2 tells you what number it represents.
The 2 in 200_{ten} means 2 —?—.
a. ones **b.** tens **c.** hundreds — **c.** hundreds

81. The 7 in 7_{ten} means 7 —?—.
a. ones **b.** tens **c.** hundreds — **a.** ones

82. The 7 in 70_{ten} means 7 —?—.
a. ones **b.** tens **c.** hundreds — **b.** tens

7	0	0
HUNDREDS	TENS	ONES

83. And the 7 in 700 $_{ten}$ means 7 ——?——.

  a. ones b. tens c. hundreds

c. hundreds

84. In base ten, the 7 in 70 $_{ten}$ is ——?—— times the 7 in 7 $_{ten}$.

  a. one b. ten c. one hundred

b. ten

85. And the 7 in 700 $_{ten}$ is ——?—— times the 7 in 70 $_{ten}$.

  a. one b. ten c. one hundred

b. ten

86. In base ten, the 4 in 40 $_{ten}$ is ________ times the 4 in 4 $_{ten}$.

ten

87. And the 6 in 600 $_{ten}$ is ________ times the 6 in 60 $_{ten}$.

ten

88. Now, let's look at base five. You know that 2 $_{five}$ means 2 ones. And 20 $_{five}$ means 2 ________.

fives

89. You can see that in base five the 2 in 20 $_{five}$ is ——?—— times the 2 in 2 $_{five}$.

  a. one b. five c. ten

b. five

90. You know that 4 $_{five}$ means 4 ________. And 40 $_{five}$ means 4 ________.

ones

fives

91. Then in base five the 4 in 40_{five} is ________ times the 4 in 4_{five}.

five

92. The 4 in 400_{five} is five times the 4 in 40_{five}. And since 40_{five} means 4 fives, we can say that 400_{five} means 4 —?—.

a. fives **b.** twenty-fives **c.** hundreds

b. twenty-fives

93. Since the 3 in 300_{five} is five times the 3 in 30_{five}, we can say that 300_{five} means 3 —?—.

a. fives **b.** twenty-fives **c.** hundreds

b. twenty-fives

94. You can see that the 3 in 3000_{five} is five times the 3 in 300_{five}. And since 300_{five} means 3 twenty-fives, we can say that 3000_{five} means 3 —?—.

a. twenty-fives **b.** one hundred twenty-fives **c.** thousands

b. one hundred twenty-fives

95. We can use the same idea when we talk about numerals in other bases. For example, in base four, 20_{four} means 2 ________.

fours

2	0	0
SIXTEENS	FOURS	ONES

96. Since the 2 in the numeral 200_{four} is four times the 2 in 20_{four}, you can see that 200_{four} means 2 —?—.

a. fours **b.** sixteens **c.** hundreds

b. sixteens

97. And since the 2 in 2000_{four} is ________ times the 2 in 200_{four}, 2000_{four} means 2 —?—.

four

a. sixteens **b.** sixty-fours **c.** thousands

b. sixty-fours

THOUSANDS	HUNDREDS	TENS	ONES
4	7	2	1

98. In base ten, the numeral 7 in 4721_{ten} means 7 —?—.

a. tens b. hundreds c. thousands

b. hundreds

99. In base five, the numeral 4 in 1432_{five} tells you how many —?— you have.

a. fives b. twenty-fives c. one hundred twenty-fives

b. twenty-fives

100. In base four, the 2 in 132_{four} tells you how many —?— there are.

a. ones b. fours c. sixteens

a. ones

101. In base seven, the 3 in 134_{seven} tells you how many —?— you have.

a. ones b. sevens c. forty-nines

b. sevens

102. The numeral 3 in 347_{eight} represents —?—.

a. 3 ones b. 3 eights c. 3 sixty-fours

c. 3 sixty-fours

TWENTY-FIVES	FIVES	ONES
1	4	2

103. The numeral 4 in 142_{five} tells you that there are —?—.

a. 4 ones b. 4 fives c. 4 twenty-fives

b. 4 fives

EIGHTY-ONES	NINES	ONES
4	4	2

104. The numeral 2 in 442 $_{nine}$ represents —?—.

a. 2 ones b. 2 nines c. 2 tens

a. 2 ones

105. When you see 10 $_{six}$, you know that this means —?—.

a. 1 one b. 1 six c. 1 thirty-six

b. 1 six

106. When you see 10 $_{eight}$, you know that this means —?—.

a. 1 one b. 1 eight c. 1 sixty-four

b. 1 eight

107. On the other hand, 100 $_{six}$ means —?—.

a. 1 one b. 1 six c. 1 thirty-six

c. 1 thirty-six

108. And 100 $_{eight}$ means —?—.

a. 1 one b. 1 eight c. 1 sixty-four

c. 1 sixty-four

From Base to Base

109. In base five, 10 $_{five}$ represents ______ five(s) and ______ one(s). Then, when we translate 10 $_{five}$ to base ten, we get —?—.

1, 0

a. 5 $_{ten}$ b. 10 $_{ten}$ c. 25 $_{ten}$

a. 5 $_{ten}$

110. In base seven, 100 $_{seven}$ represents ______ forty-nine(s), ______ seven(s), and ______ one(s). When we translate 100 $_{seven}$ to base ten, we find that 100 $_{seven}$ equals —?—.

1, 0

0

a. 7 $_{ten}$ b. 49 $_{ten}$ c. 100 $_{ten}$

b. 49 $_{ten}$

111. Transform each of the following into base ten:

1. $10_{\text{three}} = \_\_\_\_ _{\text{ten}}$ 3
2. $10_{\text{five}} = \_\_\_\_ _{\text{ten}}$ 5
3. $10_{\text{seven}} = \_\_\_\_ _{\text{ten}}$ 7
4. $10_{\text{six}} = \_\_\_\_ _{\text{ten}}$ 6
5. $10_{\text{eight}} = \_\_\_\_ _{\text{ten}}$ 8

112. Now try each of these:

1. $100_{\text{four}} = \_\_\_\_ _{\text{ten}}$ 16
2. $100_{\text{five}} = \_\_\_\_ _{\text{ten}}$ 25
3. $100_{\text{eight}} = \_\_\_\_ _{\text{ten}}$ 64
4. $100_{\text{seven}} = \_\_\_\_ _{\text{ten}}$ 49
5. $100_{\text{three}} = \_\_\_\_ _{\text{ten}}$ 9

113. Translate these into base ten:

1. $20_{\text{three}} = \_\_\_\_ _{\text{ten}}$ 6
2. $40_{\text{six}} = \_\_\_\_ _{\text{ten}}$ 24
3. $30_{\text{five}} = \_\_\_\_ _{\text{ten}}$ 15
4. $50_{\text{eight}} = \_\_\_\_ _{\text{ten}}$ 40
5. $70_{\text{nine}} = \_\_\_\_ _{\text{ten}}$ 63

114. Now try these:

1. $42_{\text{six}} = \_\_\_\_ _{\text{ten}}$ 26
2. $35_{\text{seven}} = \_\_\_\_ _{\text{ten}}$ 26
3. $61_{\text{eight}} = \_\_\_\_ _{\text{ten}}$ 49
4. $22_{\text{three}} = \_\_\_\_ _{\text{ten}}$ 8
5. $44_{\text{five}} = \_\_\_\_ _{\text{ten}}$ 24

115. And, finally, try these:

1. $123_{five} = \_\_\_\_\_\_\_\_ _{ten}$ 38

2. $204_{six} = \_\_\_\_\_\_\_\_ _{ten}$ 40

3. $333_{four} = \_\_\_\_\_\_\_\_ _{ten}$ 63

4. $320_{seven} = \_\_\_\_\_\_\_\_ _{ten}$ 161

5. $111_{nine} = \_\_\_\_\_\_\_\_ _{ten}$ 91

116. Now, let's suppose we want to write 9_{ten} as an equivalent numeral in base five. In the number nine, there is 1 five and 4 ones.

So we write 9_{ten} as ______ $_{five}$. 14

117. To write 12_{ten} as an equivalent numeral in base five, you have to see that in the number twelve there are ______ five(s) and ______ one(s). Then you can say that 12_{ten} is ______ $_{five}$. 2 / 2, 22

118. Changing 12_{ten} to base seven, you would think of twelve as ______ seven(s) and ______ one(s). That is, $12_{ten} =$ ______ $_{seven}$. 1, 5, 15

119. Fill in the blanks below:

1. $15_{ten} =$ ______ nine(s) + ______ one(s) 1, 6

2. $15_{ten} =$ ______ eight(s) + ______ one(s) 1, 7

3. $15_{ten} =$ ______ seven(s) + ______ one(s) 2, 1

4. $15_{ten} =$ ______ six(es) + ______ one(s) 2, 3

5. $15_{ten} =$ ______ five(s) + ______ one(s) 3, 0

6. $15_{ten} =$ ______ four(s) + ______ one(s) 3, 3

120. Now let's try transforming 15_{ten} into different bases:

1. $15_{\text{ten}} =$ ______ $_{\text{nine}}$ 16

2. $15_{\text{ten}} =$ ______ $_{\text{eight}}$ 17

3. $15_{\text{ten}} =$ ______ $_{\text{seven}}$ 21

4. $15_{\text{ten}} =$ ______ $_{\text{six}}$ 23

5. $15_{\text{ten}} =$ ______ $_{\text{five}}$ 30

6. $15_{\text{ten}} =$ ______ $_{\text{four}}$ 33

121. To change 75_{ten} to base nine, we think of seventy-five as ______ 8

nine(s) and ______ one(s). That is, $75_{\text{ten}} =$ ______ $_{\text{nine}}$. 3, 83

122. You can think of 16_{ten} as ______ six(es) and ______ one(s). 2, 4

Thus, $16_{\text{ten}} =$ ______ $_{\text{six}}$. 24

123. $28_{\text{ten}} =$ ______ $_{\text{eight}}$. 34

124. $12_{\text{ten}} =$ ______ $_{\text{four}}$. 30

125. Here are some numerals in base ten. Change each to the base indicated.

1. $49_{\text{ten}} =$ ______ $_{\text{eight}}$ 61

2. $38_{\text{ten}} =$ ______ $_{\text{seven}}$ 53

3. $6_{\text{ten}} =$ ______ $_{\text{three}}$ 20

4. $17_{\text{ten}} =$ ______ $_{\text{nine}}$ 18

5. $25_{\text{ten}} =$ ______ $_{\text{seven}}$ 34

126. To write 83_{ten} as an equivalent numeral in base five, you write

$83_{ten} = 3$ twenty-fives + ______ five(s) + ______ one(s). 1, 3

That is, $83_{ten} =$ ______ $_{five}$. 313

127. $71_{ten} = 2$ twenty-fives + ______ five(s) + ______ one(s). 4, 1

So, $71_{ten} =$ ______ $_{five}$. 241

128. Change each of the following numerals from base ten to base five:

1. $88_{ten} =$ ______ $_{five}$ 323
2. $47_{ten} =$ ______ $_{five}$ 142
3. $37_{ten} =$ ______ $_{five}$ 122
4. $120_{ten} =$ ______ $_{five}$ 440
5. $8_{ten} =$ ______ $_{five}$ 13

Try Your Hand

To find out how successful your work so far has been, answer each question below. The most difficult questions are marked with an asterisk (*). After you have finished answering these questions, you may check your answers on page 168.

1. Suppose your mother sent you to the store for the following items. How many of each would you buy?

a. 10_{five} pounds of potatoes
b. 21_{three} light bulbs
c. 100_{six} eggs
d. 2_{nine} quarts of ice cream
e. 22_{three} bars of candy
f. 23_{four} bars of soap
g. 44_{eight} cans of soup
h. 33_{four} pounds of sugar
i. 70_{nine} rolls of wax paper
j. 132_{four} jars of peanut butter

2. Transform 21_{ten} into each of the following bases:

a. base three
b. base four
c. base five
d. base six
e. base seven
f. base eight
g. base nine
*h. base twelve
*i. base fifteen
*j. base twenty-one

3. Answer each of the following questions:

   a. In base five, how many toes do you have?
   b. In base six, how many months are in a year?
   c. In base seven, how many feet are there in a yard?
   d. In base nine, how many fingers do you have?
   e. In base four, how many days are in a week?
   f. In base three, how many inches are there in a foot?
   *g. In base twelve, how many hours are there in a day?
   *h. In base twenty, how many days were there in February, 1957?
   *i. In base thirty, how many noses do you have?
   *j. In base fifty, how many states are there in the United States?

4. Each numeral below has been transformed to base ten for you. Tell the base of each numeral.

   a. $10_{?} = 8_{\text{ten}}$
   b. $100_{?} = 36_{\text{ten}}$
   c. $20_{?} = 14_{\text{ten}}$
   d. $11_{?} = 9_{\text{ten}}$
   e. $14_{?} = 10_{\text{ten}}$
   f. $100_{?} = 9_{\text{ten}}$
   g. $21_{?} = 15_{\text{ten}}$
   h. $43_{?} = 23_{\text{ten}}$
   i. $103_{?} = 67_{\text{ten}}$
   *j. $15_{?} = 18_{\text{ten}}$

5. Transform each of the following into base ten:

   a. 1_{four}
   b. 2_{nine}
   c. 7_{eight}
   d. 3_{five}
   e. 4_{six}
   f. 2_{three}
   g. 6_{eight}
   h. 5_{seven}
   *i. 8_{eleven}
   *j. 6_{fifteen}

6. In each case, tell what base a person would be using if he made the following statements:

   a. I have 14 toes.
   b. The Pentagon Building has 12 sides.
   c. There are 12 innings in a regular baseball game.
   d. There are 20 ounces in a pound.
   e. There are 22 months in a year.
   f. A cat has 21 lives.
   g. A triangle has 10 sides.
   *h. There are 20 seconds in a minute.
   *i. I have 10 feet.
   *j. Georgia was one of the 11 original colonies.

7. Transform these base eight numerals into base ten.

a. 3_{eight}
b. 4_{eight}
c. 5_{eight}
d. 6_{eight}
e. 7_{eight}
f. 10_{eight}
g. 11_{eight}
h. 12_{eight}
i. 20_{eight}
j. 100_{eight}

*8. Transform each of the following numerals into base ten.

a. $30_{\text{twenty-five}}$
b. 21_{thirty}
c. $36_{\text{thirty-five}}$
d. 40_{forty}
e. 100_{twenty}
f. 110_{twenty}
g. 114_{twenty}
h. 200_{twenty}
i. 316_{twenty}
j. $100_{\text{twenty-five}}$

*9. Here's a chance to show how well you know various number bases. Follow the instructions below:

a. In base five, count by twos from 1_{five} to 21_{five}.
b. In base six, count by twos from 53_{six} to 105_{six}.
c. In base nine, count backwards by twos from 17_{nine} to 6_{nine}.
d. In base three, count by twos from 1_{three} to 102_{three}.
e. In base eight, count by fours from 1_{eight} to 35_{eight}.
f. In base four, count backwards by fours from 100_{four} to 0_{four}.
g. In base ten, count by sevens from 33_{ten} to 61_{ten}.
h. In base six, count by threes from 5_{six} to 32_{six}.
i. In base ten, count by fives from 21_{ten} to 56_{ten}.
j. In base three, count by threes from 2_{three} to 202_{three}.

THINK IT OVER

■ If you like questions that force you to think hard before answering, you'll enjoy these. Don't be discouraged if some of the questions have a number of possible answers and you come up with answers that are different from those of your classmates.

1. Why don't we use the numeral 5 in base five? What digits would you use in base two? Base twelve?
2. Many centuries ago, the Maya Indians of Mexico and Central America used a system of numerals involving base twenty. Can you think of a reason for their choice of twenty as a base? Why do you think we use base ten in everyday life?
3. What features do the base seven system and the base five system have in common? In what ways are they different?
*4. An even number is a number divisible by 2. In base ten, you can easily check to find out whether a number is even or not—the numeral must end in 0, 2, 4, 6, or 8. What is a quick test to check whether a number in base eight is even? What is a quick test to check whether a number in base five is even?

2 A Look at Addition

DOES $4 + 3 = 10$?

Now that you have learned something about expressing numbers in various bases, what do you do with them? You can begin by *adding* them.

At the beginning of this book you were told that $4 + 3$ is sometimes equal to 10, sometimes equal to 11, and sometimes equal to 12. How can this be so? How can $4 + 3 = 10$, 11, and 12? Is it still true that $4 + 3 = 7$? Can $4 + 3 = 8$?

As you go through this chapter, you will find that you will be able to answer each of these questions. The answers, you will find, have a lot to do with the *base* in which the numerals are written.

Adding "Below the Base"

1. You know how to add in base ten. You know, for example, that

$14_{\text{ten}} + 7_{\text{ten}} = \_\_\_\_\__{\text{ten}}$. 21

2. In base ten, $1_{\text{ten}} + 8_{\text{ten}} = \_\_\_\__{\text{ten}}$. The *addends* are 1_{ten} and 8_{ten}, 9

and the *sum*, of course, is $\_\_\_\__{\text{ten}}$. 9

3. Now we are going to take a look at addition in other number bases. First, we will deal with examples in which the sum is the same as it would be in base ten.

In base five, $0_{\text{five}} + 1_{\text{five}} =$ ____ five. 1

4. $2_{\text{five}} + 2_{\text{five}} =$ ____ five. 4

5. The sum of 3_{five} and 1_{five} is ____ five. 4

6. Write the sum in base five for each of the following:

1. $1_{\text{five}} + 0_{\text{five}} =$ ____ five 1
2. $2_{\text{five}} + 1_{\text{five}} =$ ____ five 3
3. $0_{\text{five}} + 4_{\text{five}} =$ ____ five 4
4. $1_{\text{five}} + 3_{\text{five}} =$ ____ five 4
5. $1_{\text{five}} + 1_{\text{five}} =$ ____ five 2

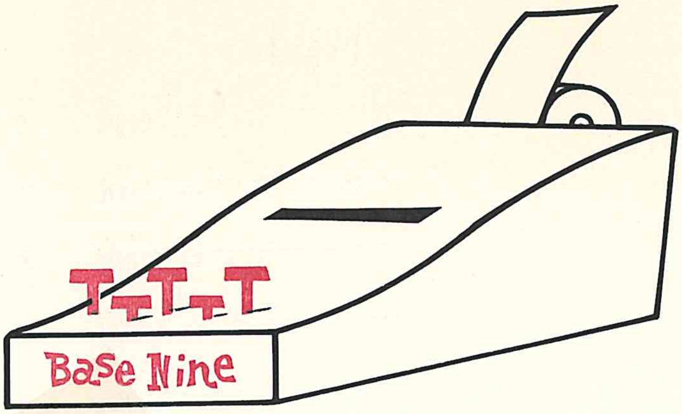
7. You can also find the sum when there are three addends. For example, $1_{\text{five}} + 1_{\text{five}} + 2_{\text{five}} =$ ____ five. 4

8. And $3_{\text{five}} + 1_{\text{five}} + 0_{\text{five}} =$ ____ five. 4

9. Now let's take a look at addition in another base. (We are still dealing with examples in which the process of adding is the same as it is in base ten.)

In base nine, $2_{\text{nine}} + 6_{\text{nine}} =$ ____ nine. 8

10. $1_{\text{nine}} + 2_{\text{nine}} + 3_{\text{nine}} =$ ____ nine. 6



11. Here are some more problems in base nine:

1. $7_{\text{nine}} + 0_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 7
2. $2_{\text{nine}} + 2_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 4
3. $5_{\text{nine}} + 3_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 8
4. $1_{\text{nine}} + 1_{\text{nine}} + 4_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 6
5. $2_{\text{nine}} + 2_{\text{nine}} + 2_{\text{nine}} + 2_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 8

12. You remember that 10_{five} means 1 —?—.

a. five **b.** ten **c.** one

a. five

13. Then $10_{\text{five}} + 10_{\text{five}}$ means 1 five plus 1 five, which is —?— five(s).

a. 1 **b.** 2 **c.** 5

b. 2

14. Thus, $10_{\text{five}} + 10_{\text{five}} = \_\_\_\__{\text{five}}$. — 20

15. And $20_{\text{five}} + 10_{\text{five}} = \_\_\_\__{\text{five}}$. — 30

16. You know that

$$3_{\text{five}} + 1_{\text{five}} = 4_{\text{five}}.$$

So you can see that

$$30_{\text{five}} + 10_{\text{five}} = \_\_\_\__{\text{five}}.$$

40

17. $20_{\text{five}} + 20_{\text{five}} =$ ______ five. 40

18. In base five, 200_{five} means ——?—— twenty-five(s).

a. 1 b. 2 c. 3 b. 2

19. Then $200_{\text{five}} + 200_{\text{five}}$ means 2 twenty-fives plus 2 twenty-fives.

Altogether, there are ______ twenty-fives. 4

20. So $200_{\text{five}} + 200_{\text{five}} =$ ______ five. 400

21. Here are more addition problems in base five:

1. $10_{\text{five}} + 10_{\text{five}} =$ ______ five 20
2. $30_{\text{five}} + 10_{\text{five}} =$ ______ five 40
3. $10_{\text{five}} + 20_{\text{five}} =$ ______ five 30
4. $20_{\text{five}} + 20_{\text{five}} =$ ______ five 40
5. $10_{\text{five}} + 10_{\text{five}} + 20_{\text{five}} =$ ______ five 40

22. Try these addition problems in base five:

1. $100_{\text{five}} + 100_{\text{five}} =$ ______ five 200
2. $300_{\text{five}} + 100_{\text{five}} =$ ______ five 400
3. $100_{\text{five}} + 200_{\text{five}} + 100_{\text{five}} =$ ______ five 400
4. $2000_{\text{five}} + 1000_{\text{five}} =$ ______ five 3000
5. $1000_{\text{five}} + 1000_{\text{five}} + 1000_{\text{five}} =$ ______ five 3000

23. Let's change to another base. Each of the following examples is in base nine. Find the sum in each case.

1. $20_{\text{nine}} + 20_{\text{nine}} =$ ______ $_{\text{nine}}$ — 40

2. $30_{\text{nine}} + 50_{\text{nine}} =$ ______ $_{\text{nine}}$ — 80

3. $100_{\text{nine}} + 700_{\text{nine}} =$ ______ $_{\text{nine}}$ — 800

4. $2000_{\text{nine}} + 3000_{\text{nine}} =$ ______ $_{\text{nine}}$ — 5000

5. $3000_{\text{nine}} + 1000_{\text{nine}} + 2000_{\text{nine}} =$ ______ $_{\text{nine}}$ — 6000

24. Now here are some examples in various bases. Try these:

1. $2_{\text{six}} + 3_{\text{six}} =$ ______ $_{\text{six}}$ — 5

2. $3_{\text{seven}} + 2_{\text{seven}} =$ ______ $_{\text{seven}}$ — 5

3. $5_{\text{nine}} + 2_{\text{nine}} =$ ______ $_{\text{nine}}$ — 7

4. $40_{\text{eight}} + 20_{\text{eight}} =$ ______ $_{\text{eight}}$ — 60

5. $100_{\text{three}} + 100_{\text{three}} =$ ______ $_{\text{three}}$ — 200

6. $700_{\text{nine}} + 100_{\text{nine}} =$ ______ $_{\text{nine}}$ — 800

7. $200_{\text{six}} + 300_{\text{six}} =$ ______ $_{\text{six}}$ — 500

8. $400_{\text{ten}} + 500_{\text{ten}} =$ ______ $_{\text{ten}}$ — 900

9. $3000_{\text{eight}} + 2000_{\text{eight}} =$ ______ $_{\text{eight}}$ — 5000

10. $2000_{\text{nine}} + 6000_{\text{nine}} =$ ______ $_{\text{nine}}$ — 8000

25. In base seven, 30_{seven} means 3 —?—.

a. ones **b.** sevens **c.** forty-nines — **b.** sevens

26. And 6_{seven} means 6 —?—.

a. ones **b.** sevens **c.** forty-nines — **a.** ones

27. Then $30_{\text{seven}} + 6_{\text{seven}}$ means ______ seven(s) plus ______ one(s). — 3, 6

28. So $30_{\text{seven}} + 6_{\text{seven}} =$ ______ $_{\text{seven}}$. 36

29. Here's another one. In base five, 30_{five} means 3 —?—.

a. ones b. fives c. twenty-fives b. fives

30. And 3_{five} means 3 —?—.

a. ones b. fives c. twenty-fives a. ones

31. $30_{\text{five}} + 3_{\text{five}} =$ ______ $_{\text{five}}$. 33

32. In base eight, $40_{\text{eight}} + 6_{\text{eight}}$ means ______ eight(s) plus ______ 4, 6

one(s). So $40_{\text{eight}} + 6_{\text{eight}} =$ ______ $_{\text{eight}}$. 46

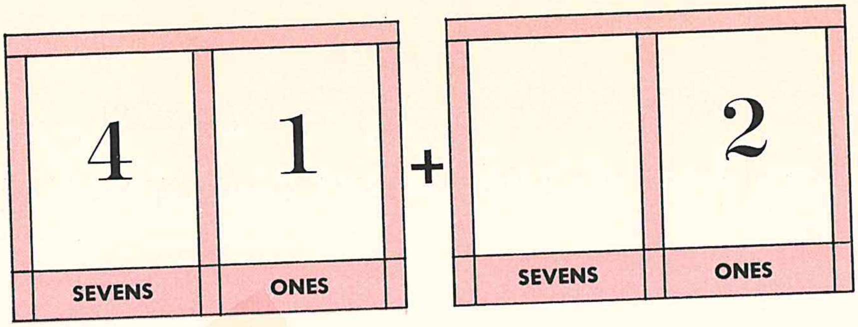
33. In base three, $20_{\text{three}} + 2_{\text{three}} =$ ______ $_{\text{three}}$. 22

34. In base five, $200_{\text{five}} + 30_{\text{five}} =$ ______ $_{\text{five}}$. 230

35. And $320_{\text{five}} + 4_{\text{five}} =$ ______ $_{\text{five}}$. 324

36. Fill in the sum in each of the following:

1. $220_{\text{five}} + 3_{\text{five}} =$ ______ $_{\text{five}}$ 223
2. $40_{\text{six}} + 5_{\text{six}} =$ ______ $_{\text{six}}$ 45
3. $1000_{\text{eight}} + 300_{\text{eight}} + 10_{\text{eight}} =$ ______ $_{\text{eight}}$ 1310
4. $2000_{\text{nine}} + 100_{\text{nine}} + 30_{\text{nine}} + 6_{\text{nine}} =$ ______ $_{\text{nine}}$ 2136
5. $1000_{\text{ten}} + 400_{\text{ten}} + 20_{\text{ten}} + 8_{\text{ten}} =$ ______ $_{\text{ten}}$ 1428



37. In base seven, 41_{seven} means ______ seven(s) and ______ one(s). 4, 1

And 2_{seven} means ______ one(s). 2

38. Then, $41_{\text{seven}} + 2_{\text{seven}}$ means ______ seven(s) and ______ one(s) 4, 1

plus ______ one(s). 2

39. Altogether, then, there are ______ seven(s) and ______ one(s). 4, 3

40. So $41_{\text{seven}} + 2_{\text{seven}} =$ ______ $_{\text{seven}}$. 43

41. In base five, $21_{\text{five}} + 3_{\text{five}} =$ ______ $_{\text{five}}$. 24

42. And in base seven, $412_{\text{seven}} + 31_{\text{seven}} =$ ______ $_{\text{seven}}$. 443

43. Fill in the sum in each of the following:

1. $22_{\text{five}} + 2_{\text{five}} =$ ______ $_{\text{five}}$ 24
2. $101_{\text{six}} + 21_{\text{six}} =$ ______ $_{\text{six}}$ 122
3. $456_{\text{nine}} + 22_{\text{nine}} =$ ______ $_{\text{nine}}$ 478
4. $307_{\text{eight}} + 120_{\text{eight}} =$ ______ $_{\text{eight}}$ 427
5. $418_{\text{ten}} + 321_{\text{ten}} =$ ______ $_{\text{ten}}$ 739

Adding "To the Base"

44. The addition examples you've seen so far have been just the same as base ten addition examples. You saw, for example, that $2_{\text{five}} + 2_{\text{five}} = 4_{\text{five}}$. You saw also that $101_{\text{six}} + 21_{\text{six}} = 122_{\text{six}}$. In base ten, $2_{\text{ten}} + 2_{\text{ten}} = 4_{\text{ten}}$, and $101_{\text{ten}} + 21_{\text{ten}} = 122_{\text{ten}}$.

Now you are going to look at addition examples which are very different from base ten examples.

In base five, $2_{\text{five}} + 3_{\text{five}}$ gives you ______ five(s) and 0 ones. You remember that in base five this is written as 10_{five}. Thus, $2_{\text{five}} + 3_{\text{five}} =$ ——?——. 1

a. 10_{five} **b.** 5_{five} **c.** 100_{five} **a.** 10_{five}

45. $4_{\text{five}} + 1_{\text{five}} =$ ______ $_{\text{five}}$. 10

46. In base nine, $8_{\text{nine}} + 1_{\text{nine}}$ gives you ______ nine(s) and ______ 1, 0

one(s). So $8_{\text{nine}} + 1_{\text{nine}} =$ ______ $_{\text{nine}}$. 10

47. $3_{\text{nine}} + 6_{\text{nine}} =$ ______ $_{\text{nine}}$. 10

48. And in base seven, $4_{\text{seven}} + 3_{\text{seven}} =$ ______ $_{\text{seven}}$. 10

49. Try each of the following addition examples:

1. $4_{\text{six}} + 2_{\text{six}} =$ ______ $_{\text{six}}$ 10
2. $6_{\text{eight}} + 2_{\text{eight}} =$ ______ $_{\text{eight}}$ 10
3. $3_{\text{six}} + 3_{\text{six}} =$ ______ $_{\text{six}}$ 10
4. $1_{\text{four}} + 3_{\text{four}} =$ ______ $_{\text{four}}$ 10
5. $2_{\text{three}} + 1_{\text{three}} =$ ______ $_{\text{three}}$ 10

50. In each of the following cases, find the missing addend. Notice that the sum in each case is the base (written as 10).

1. $2_{\text{nine}} + \_\_\_\__{\text{nine}} = 10_{\text{nine}}$ — 7

2. $1_{\text{five}} + \_\_\_\__{\text{five}} = 10_{\text{five}}$ — 4

3. $5_{\text{eight}} + \_\_\_\__{\text{eight}} = 10_{\text{eight}}$ — 3

4. $4_{\text{seven}} + \_\_\_\__{\text{seven}} = 10_{\text{seven}}$ — 3

5. $2_{\text{six}} + \_\_\_\__{\text{six}} = 10_{\text{six}}$ — 4

51. Find the missing addend in each of the following cases:

1. $2_{\text{five}} + 1_{\text{five}} + \_\_\_\__{\text{five}} = 10_{\text{five}}$ — 2

2. $3_{\text{nine}} + 2_{\text{nine}} + \_\_\_\__{\text{nine}} = 10_{\text{nine}}$ — 4

3. $2_{\text{seven}} + \_\_\_\__{\text{seven}} + 3_{\text{seven}} = 10_{\text{seven}}$ — 2

4. $\_\_\_\__{\text{eight}} + 1_{\text{eight}} + 3_{\text{eight}} = 10_{\text{eight}}$ — 4

5. $1_{\text{three}} + 1_{\text{three}} + \_\_\_\__{\text{three}} = 10_{\text{three}}$ — 1

52. Earlier in this chapter, you saw that

$$2_{\text{five}} + 1_{\text{five}} = 3_{\text{five}}.$$

You were then able to say that

$$200_{\text{five}} + 100_{\text{five}} = \_\_\_\_\_\__{\text{five}}.$$ 300

53. Now since

$$4_{\text{five}} + 1_{\text{five}} = 10_{\text{five}},$$

you can see that

$$40_{\text{five}} + 10_{\text{five}} = \text{—?—}.$$

a. 10_{five} b. 100_{five} c. 50_{five} — b. 100_{five}

54. 30_{five} means 3 fives. And 20_{five} means 2 fives. Then, 3 fives and 2 fives equal 5 fives , or ______ twenty-five(s). — 1

Thus, in base five, $30_{\text{five}} + 20_{\text{five}} = \text{—?—}$.

a. 100_{five} b. 50_{five} c. 10_{five} — a. 100_{five}

55. In base seven, $30_{\text{seven}} + 40_{\text{seven}} =$ ________ seven. 100

56. And $20_{\text{seven}} + 50_{\text{seven}} =$ ________ seven. 100

57. Since

$$3_{\text{nine}} + 6_{\text{nine}} = 10_{\text{nine}},$$

you can see that

$$30_{\text{nine}} + 60_{\text{nine}} = \_\_\_\_\_\_\_\_ {}_{\text{nine}}.$$ 100

58. In base four:

$$3_{\text{four}} + 1_{\text{four}} = 10_{\text{four}}.$$
$$30_{\text{four}} + 10_{\text{four}} = 100_{\text{four}}.$$
$$300_{\text{four}} + 100_{\text{four}} = \_\_\_\_\_\_\_\_ {}_{\text{four}}.$$ 1000

59. In base eight, $400_{\text{eight}} + 400_{\text{eight}} =$ ________ eight. 1000

60. And in base ten, $800_{\text{ten}} + 200_{\text{ten}} =$ ________ ten. 1000

61. Find the sum in each of the following cases:

1. $30_{\text{seven}} + 40_{\text{seven}} =$ ________ seven 100
2. $40_{\text{nine}} + 50_{\text{nine}} =$ ________ nine 100
3. $500_{\text{seven}} + 200_{\text{seven}} =$ ________ seven 1000
4. $6000_{\text{ten}} + 4000_{\text{ten}} =$ ________ ten 10000
5. $2000_{\text{three}} + 1000_{\text{three}} =$ ________ three 10000

2. Now find the missing addends:

1. $300_{\text{nine}} + \_\_\_\_\_\_\_\__{\text{nine}} = 1000_{\text{nine}}$ — 600

2. $700_{\text{ten}} + \_\_\_\_\_\_\_\__{\text{ten}} = 1000_{\text{ten}}$ — 300

3. $500_{\text{seven}} + \_\_\_\_\_\_\_\__{\text{seven}} = 1000_{\text{seven}}$ — 200

4. $\_\_\_\_\_\_\_\__{\text{eight}} + 4000_{\text{eight}} = 10000_{\text{eight}}$ — 4000

5. $\_\_\_\_\_\_\_\__{\text{three}} + 200_{\text{three}} = 1000_{\text{three}}$ — 100

3. We know that $3_{\text{five}} + 2_{\text{five}} = 10_{\text{five}}$. We use this fact when we say $13_{\text{five}} + 2_{\text{five}} = 20_{\text{five}}$. We know that 13_{five} means 1 five and 3 ones. And 2_{five} simply means 2 ones. When we add, then, we have

______ five(s) and 0 ones. — 2

4. $11_{\text{five}} + 4_{\text{five}} =$ —?— $_{\text{five}}$.

a. 15_{five} b. 20_{five} c. 101_{five} — b. 20_{five}

5. You know that $1_{\text{five}} + 4_{\text{five}} = 10_{\text{five}}$. And you also know that $11_{\text{five}} + 4_{\text{five}} = 20_{\text{five}}$.

Then, of course, $21_{\text{five}} + 4_{\text{five}} = \_\_\_\_\_\__{\text{five}}$. — 30

6. The same principle holds true in base seven. For example:

$$5_{\text{seven}} + 2_{\text{seven}} = 10_{\text{seven}}.$$

$15_{\text{seven}} + 2_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$. — 20

$25_{\text{seven}} + 2_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$. — 30

7. In base seven, find the sums:

1. $24_{\text{seven}} + 3_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 30

2. $14_{\text{seven}} + 3_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 20

3. $54_{\text{seven}} + 3_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 60

8. Find the sum in each case below:

1. $33_{\text{seven}} + 4_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 40
2. $12_{\text{seven}} + 5_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 20
3. $46_{\text{seven}} + 1_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 50
4. $24_{\text{seven}} + 3_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 30
5. $51_{\text{seven}} + 6_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ — 60

9. Now find the sum in each case here:

1. $18_{\text{nine}} + 1_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}$ — 20
2. $36_{\text{ten}} + 4_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}$ — 40
3. $12_{\text{three}} + 1_{\text{three}} = \_\_\_\_\_\__{\text{three}}$ — 20
4. $22_{\text{four}} + 2_{\text{four}} = \_\_\_\_\_\__{\text{four}}$ — 30
5. $66_{\text{eight}} + 2_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}$ — 70

0. Find the missing addend in each of the following:

1. $22_{\text{nine}} + \_\_\_\_\_\__{\text{nine}} = 30_{\text{nine}}$ — 7
2. $13_{\text{five}} + \_\_\_\_\_\__{\text{five}} = 20_{\text{five}}$ — 2
3. $37_{\text{ten}} + \_\_\_\_\_\__{\text{ten}} = 40_{\text{ten}}$ — 3
4. $67_{\text{eight}} + \_\_\_\_\_\__{\text{eight}} = 70_{\text{eight}}$ — 1
5. $22_{\text{six}} + \_\_\_\_\_\__{\text{six}} = 30_{\text{six}}$ — 4

1. Now, in base five, let's extend the principle we are using. Suppose we add 44_{five} and 1_{five}. Altogether, we have 4 fives and 5 ones. But this is the same as 5 fives. And 5 fives make ______ twenty-five(s). Thus, $44_{\text{five}} + 1_{\text{five}} =$ ——?——. — 1

a. 45_{five} **b.** 100_{five} **c.** 50_{five} — **b.** 100_{five}

2. Let's add 63_{seven} and 4_{seven}. Altogether, we have 6 sevens and 7 ones. But this is the same as 7 sevens. In base seven, then, $63_{\text{seven}} + 4_{\text{seven}} =$ ——?——.

a. 67_{seven} **b.** 70_{seven} **c.** 100_{seven} — **c.** 100_{seven}

In base five, $42_{\text{five}} + 3_{\text{five}} =$ ________ $_{\text{five}}$. 100

In base seven, $61_{\text{seven}} + 6_{\text{seven}} =$ ________ $_{\text{seven}}$. 100

In base four, $32_{\text{four}} + 2_{\text{four}} =$ ________ $_{\text{four}}$. 100

Find each of the following sums:

1. $33_{\text{four}} + 1_{\text{four}} =$ ________ $_{\text{four}}$ 100
2. $82_{\text{nine}} + 7_{\text{nine}} =$ ________ $_{\text{nine}}$ 100
3. $22_{\text{three}} + 1_{\text{three}} =$ ________ $_{\text{three}}$ 100
4. $54_{\text{six}} + 2_{\text{six}} =$ ________ $_{\text{six}}$ 100
5. $62_{\text{seven}} + 5_{\text{seven}} =$ ________ $_{\text{seven}}$ 100

. Find each of the missing addends:

1. $64_{\text{seven}} +$ ____ $_{\text{seven}} = 100_{\text{seven}}$ 3
2. $21_{\text{three}} +$ ____ $_{\text{three}} = 100_{\text{three}}$ 2
3. $94_{\text{ten}} +$ ____ $_{\text{ten}} = 100_{\text{ten}}$ 6
4. $73_{\text{eight}} +$ ____ $_{\text{eight}} = 100_{\text{eight}}$ 5
5. $52_{\text{six}} +$ ____ $_{\text{six}} = 100_{\text{six}}$ 4

. In base five:

$$4_{\text{five}} + 1_{\text{five}} = 10_{\text{five}}$$

$$44_{\text{five}} + 1_{\text{five}} = 100_{\text{five}}$$

$444_{\text{five}} + 1_{\text{five}} =$ ________ $_{\text{five}}$ 1000

. In base seven, $664_{\text{seven}} + 3_{\text{seven}} =$ ________ $_{\text{seven}}$. 1000

80. And in base three, $221_{three} + 2_{three} =$ ________ $_{three}$. 1000

31. In base seven, $6663_{seven} + 4_{seven} =$ —?—.

a. 666_{seven} b. 1000_{seven} c. 10000_{seven} c. 10000_{seven}

2. And in base nine, $8882_{nine} + 7_{nine} =$ ________ $_{nine}$. 10000

3. Find each of the following sums:

1. $441_{five} + 4_{five} =$ ________ $_{five}$ 1000
2. $222_{three} + 1_{three} =$ ________ $_{three}$ 1000
3. $3331_{four} + 3_{four} =$ ________ $_{four}$ 10000
4. $7774_{eight} + 4_{eight} =$ ________ $_{eight}$ 10000
5. $88884_{nine} + 5_{nine} =$ ________ $_{nine}$ 100000

4. Find each of the missing addends:

1. $222_{three} +$ ____ $_{three} = 1000_{three}$ 1
2. $332_{four} +$ ____ $_{four} = 1000_{four}$ 2
3. $4441_{five} +$ ____ $_{five} = 10000_{five}$ 4
4. $662_{seven} +$ ____ $_{seven} = 1000_{seven}$ 5
5. $992_{ten} +$ ____ $_{ten} = 1000_{ten}$ 8

. You remember that

$$34_{five} + 1_{five} = 40_{five}.$$

Then we can say that

$$340_{five} + 10_{five} = \text{\_\_\_\_\_\_\_\_}_{five}.$$ 400

In base four:

$$23_{four} + 1_{four} = 30_{four}$$
$$230_{four} + 10_{four} = 300_{four}$$
$$2300_{four} + 100_{four} = \text{\_\_\_\_\_\_\_\_}_{four}$$ 3000

$22^{00} + 4^{00} = ??^{00}$

87. In base six:

$22_{six} + 4_{six} = 30_{six}$

$220_{six} + 40_{six} =$ ______ $_{six}$ 300

$2200_{six} + 400_{six} =$ ______ $_{six}$ 3000

88. In base seven:

$62_{seven} + 5_{seven} = 100_{seven}$

$620_{seven} + 50_{seven} = 1000_{seven}$

$6200_{seven} + 500_{seven} =$ ______ $_{seven}$ 10000

89. Find the sum in each of the following cases:

1. $130_{seven} + 40_{seven} =$ ______ $_{seven}$ 200
2. $220_{five} + 30_{five} =$ ______ $_{five}$ 300
3. $550_{six} + 10_{six} =$ ______ $_{six}$ 1000
4. $990_{ten} + 10_{ten} =$ ______ $_{ten}$ 1000
5. $640_{nine} + 50_{nine} =$ ______ $_{nine}$ 700

90. Now find the missing addends:

1. $440_{five} +$ ______ $_{five} = 1000_{five}$ 10
2. $320_{six} +$ ______ $_{six} = 400_{six}$ 40
3. ______ $_{eight} + 2640_{eight} = 2700_{eight}$ 40
4. ______ $_{ten} + 140_{ten} = 200_{ten}$ 60
5. ______ $_{five} + 1230_{five} = 1300_{five}$ 20

Adding "Beyond the Base"

1. So far, in studying addition, we have seen two situations. The first involved addition examples which were practically the same as base ten. We saw, for example, that $3_{\text{five}} + 1_{\text{five}} = 4_{\text{five}}$ and $456_{\text{nine}} + 22_{\text{nine}} = 478_{\text{nine}}$.

The second situation involved examples very much different from base ten. We saw, for example, that $2_{\text{four}} + 2_{\text{four}} = 10_{\text{four}}$ and $64_{\text{seven}} + 3_{\text{seven}} = 100_{\text{seven}}$.

Now we are going to take a look at a third situation. Suppose we want to find the sum of 4_{five} and 2_{five}. There are many ways of finding this sum. The method we'll choose is related to the methods we saw earlier in this chapter.

We want to find the sum of 4_{five} and 2_{five}. Instead of adding 2_{five} to 4_{five} right away, we will add 1_{five} to 4_{five} to get to the base: $4_{\text{five}} + 1_{\text{five}} =$ ______ $_{\text{five}}$. 10

2. And since 2_{five} is 1 more than 1_{five}, $4_{\text{five}} + 2_{\text{five}}$ is 1 more than 10_{five}. In other words, $4_{\text{five}} + 2_{\text{five}} =$ ______ $_{\text{five}}$. 11

3. Let's try this method on another problem. Let's find the sum of 3_{five} and 4_{five}. Instead of adding 4_{five} to 3_{five} right away, we'll add 2_{five} to 3_{five}. Thus $3_{\text{five}} + 2_{\text{five}} =$ ______ $_{\text{five}}$. 10

4. Then $3_{\text{five}} + 4_{\text{five}} = 12_{\text{five}}$, because 4_{five} is —?— than 2_{five}.
   a. 2 more **b.** 2 less **c.** 3 more **a.** 2 more

5. Let's try adding 6_{nine} and 8_{nine} by this method. First, we say $6_{\text{nine}} + 3_{\text{nine}} = 10_{\text{nine}}$. Then, since 8_{nine} is 5 more than 3_{nine}, we know that $6_{\text{nine}} + 8_{\text{nine}}$ is 5 more than 10_{nine}. That is, $6_{\text{nine}} + 8_{\text{nine}} =$ ______ $_{\text{nine}}$. 15

6. In base ten, let's add 9_{ten} and 6_{ten}. First, $9_{\text{ten}} +$ ______ $_{\text{ten}} = 10_{\text{ten}}$. 1
But 6_{ten} is ______ more than 1_{ten}. And thus, $9_{\text{ten}} + 6_{\text{ten}} =$ ______ $_{\text{ten}}$. 5, 15

97. To add 7_{nine} and 8_{nine}, you can say that

$7_{\text{nine}} + 8_{\text{nine}} = 7_{\text{nine}} + 2_{\text{nine}} + \_\_\_\__{\text{nine}}.$ 6

Then

$7_{\text{nine}} + 8_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}.$ 16

98. $3_{\text{seven}} + 6_{\text{seven}} = 3_{\text{seven}} + 4_{\text{seven}} + \_\_\_\__{\text{seven}}.$ 2

$3_{\text{seven}} + 6_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}.$ 12

99. $4_{\text{eight}} + 7_{\text{eight}} = 4_{\text{eight}} + \_\_\_\__{\text{eight}} + \_\_\_\__{\text{eight}}.$ 4, 3

$4_{\text{eight}} + 7_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}.$ 13

100. For practice, complete the blanks in the following:

1. $3_{\text{five}} + 3_{\text{five}} = 3_{\text{five}} + \_\_\_\__{\text{five}} + 1_{\text{five}}$ 2
2. $8_{\text{ten}} + 7_{\text{ten}} = 8_{\text{ten}} + \_\_\_\__{\text{ten}} + 5_{\text{ten}}$ 2
3. $6_{\text{nine}} + 5_{\text{nine}} = 6_{\text{nine}} + 3_{\text{nine}} + \_\_\_\__{\text{nine}}$ 2
4. $4_{\text{eight}} + 7_{\text{eight}} = 4_{\text{eight}} + \_\_\_\__{\text{eight}} + \_\_\_\__{\text{eight}}$ 4, 3
5. $5_{\text{six}} + 5_{\text{six}} = 5_{\text{six}} + \_\_\_\__{\text{six}} + \_\_\_\__{\text{six}}$ 1, 4

101. $8_{\text{nine}} + 5_{\text{nine}} = 10_{\text{nine}} + \_\_\_\__{\text{nine}}.$ 4

$8_{\text{nine}} + 5_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}.$ 14

102. $6_{\text{eight}} + 5_{\text{eight}} = 10_{\text{eight}} + \_\_\_\__{\text{eight}}.$ 3

$6_{\text{eight}} + 5_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}.$ 13

103. $8_{\text{ten}} + 6_{\text{ten}} = 10_{\text{ten}} + \_\_\_\__{\text{ten}}.$ 4

$8_{\text{ten}} + 6_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}.$ 14

104. $4_{\text{five}} + 4_{\text{five}} = 10_{\text{five}} + \_\_\_\__{\text{five}}.$ 3

$4_{\text{five}} + 4_{\text{five}} = \_\_\_\_\_\__{\text{five}}.$ 13

105. Now let's omit the middle step in writing these problems. Do the calculation of the middle step mentally.

$$4_{\text{five}} + 3_{\text{five}} = \_\_\_\_\_\__{\text{five}}.$$ 12

106. $6_{\text{ten}} + 8_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}.$ 14

107. $8_{\text{nine}} + 5_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}.$ 14

108. $6_{\text{seven}} + 6_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}.$ 15

109. Find the sums in the following examples:

1. $3_{\text{seven}} + 6_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ 12
2. $4_{\text{eight}} + 6_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}$ 12
3. $9_{\text{ten}} + 6_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}$ 15
4. $2_{\text{three}} + 2_{\text{three}} = \_\_\_\_\_\__{\text{three}}$ 11
5. $8_{\text{nine}} + 7_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}$ 16

110. Now find the missing addend in each of the following cases:

1. $2_{\text{three}} + \_\_\_\__{\text{three}} = 11_{\text{three}}$ 2
2. $4_{\text{five}} + \_\_\_\__{\text{five}} = 13_{\text{five}}$ 4
3. $8_{\text{nine}} + \_\_\_\__{\text{nine}} = 16_{\text{nine}}$ 7
4. $\_\_\_\__{\text{ten}} + 6_{\text{ten}} = 14_{\text{ten}}$ 8
5. $\_\_\_\__{\text{six}} + 5_{\text{six}} = 14_{\text{six}}$ 5

111. In order to add 40_{five} and 40_{five}, we can look at the fact that

$$4_{\text{five}} + 4_{\text{five}} = 13_{\text{five}}.$$

Then,

$$40_{\text{five}} + 40_{\text{five}} = \_\_\_\_\_\__{\text{five}}.$$ 130

112. Since

$$5_{\text{eight}} + 7_{\text{eight}} = 14_{\text{eight}},$$

you know that

$$50_{\text{eight}} + 70_{\text{eight}} = \_\_\_\_\_\_\_\__{\text{eight}}.$$ 140

113. In base nine, $70_{\text{nine}} + 50_{\text{nine}} = \_\_\_\_\_\_\_\__{\text{nine}}$. 130

114. And in base seven, $600_{\text{seven}} + 300_{\text{seven}} = \_\_\_\_\_\_\_\__{\text{seven}}$. 1200

115. Find the sums of each of the following:

1. $40_{\text{five}} + 30_{\text{five}} = \_\_\_\_\_\_\_\__{\text{five}}$ 120
2. $30_{\text{seven}} + 50_{\text{seven}} = \_\_\_\_\_\_\_\__{\text{seven}}$ 110
3. $200_{\text{four}} + 300_{\text{four}} = \_\_\_\_\_\_\_\__{\text{four}}$ 1100
4. $600_{\text{nine}} + 500_{\text{nine}} = \_\_\_\_\_\_\_\__{\text{nine}}$ 1200
5. $400_{\text{six}} + 400_{\text{six}} = \_\_\_\_\_\_\_\__{\text{six}}$ 1200

116. In order to add 18_{nine} and 4_{nine}, we can say that

$$18_{\text{nine}} + \_\_\_\_\_\__{\text{nine}} = 20_{\text{nine}}.$$ 1

Then,

$$18_{\text{nine}} + 4_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}.$$ 23

117. Let's find the sum of 44_{seven} and 6_{seven}:

$$44_{\text{seven}} + \_\_\_\__{\text{seven}} = 50_{\text{seven}}$$ 3

$$44_{\text{seven}} + 6_{\text{seven}} = 50_{\text{seven}} + \_\_\_\__{\text{seven}}$$ 3

$$44_{\text{seven}} + 6_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$$ 53

118. Let's add 34_{five} and 4_{five}:

$$34_{\text{five}} + \_\_\_\__{\text{five}} = 40_{\text{five}}$$ 1

$$34_{\text{five}} + 4_{\text{five}} = 40_{\text{five}} + \_\_\_\__{\text{five}}$$ 3

$$34_{\text{five}} + 4_{\text{five}} = \_\_\_\_\_\__{\text{five}}$$ 43

119. In base six, $24_{six} + 3_{six} =$ ______ $_{six}$. 31

120. And in base seven, $56_{seven} + 6_{seven} =$ ______ $_{seven}$. 65

121. Find the sum in each of the following examples:

1. $26_{seven} + 5_{seven} =$ ______ $_{seven}$ 34
2. $65_{eight} + 7_{eight} =$ ______ $_{eight}$ 74
3. $124_{ten} + 9_{ten} =$ ______ $_{ten}$ 133
4. $223_{five} + 4_{five} =$ ______ $_{five}$ 232
5. $346_{nine} + 8_{nine} =$ ______ $_{nine}$ 355

122. Since you know that

$$26_{seven} + 3_{seven} = 32_{seven},$$

You can also say that

$$260_{seven} + 30_{seven} = \text{\_\_\_\_\_\_}_{seven}.$$ 320

123. Find the sums in each of the following:

1. $40_{five} + 40_{five} =$ ______ $_{five}$ 130
2. $70_{eight} + 30_{eight} =$ ______ $_{eight}$ 120
3. $460_{seven} + 50_{seven} =$ ______ $_{seven}$ 540
4. $980_{ten} + 70_{ten} =$ ______ $_{ten}$ 1050
5. $250_{six} + 40_{six} =$ ______ $_{six}$ 330

124. In base five, $44_{five} + 1_{five} = 100_{five}$. Then it is easy to see that

$44_{five} + 3_{five} =$ ______ $_{five}$. 102

125. In base six, $54_{\text{six}} + 2_{\text{six}} = 100_{\text{six}}$. Then you know that

$54_{\text{six}} + 5_{\text{six}} =$ ________ six. 103

126. And in base eight, $76_{\text{eight}} + 2_{\text{eight}} = 100_{\text{eight}}$. You know, then,

that $76_{\text{eight}} + 7_{\text{eight}} =$ ________ eight. 105

127. $42_{\text{five}} + 4_{\text{five}} =$ ________ five. 101

128. $143_{\text{five}} + 4_{\text{five}} =$ ________ five. 202

129. Find the sums in each of the following cases:

1. $22_{\text{three}} + 2_{\text{three}} =$ ________ three 101
2. $84_{\text{nine}} + 7_{\text{nine}} =$ ________ nine 102
3. $98_{\text{ten}} + 8_{\text{ten}} =$ ________ ten 106
4. $144_{\text{five}} + 4_{\text{five}} =$ ________ five 203
5. $555_{\text{six}} + 5_{\text{six}} =$ ________ six 1004

130. You know that $12_{\text{five}} + 4_{\text{five}} = 21_{\text{five}}$. Then you can see that

$12_{\text{five}} + 14_{\text{five}} = 31_{\text{five}}$, and $12_{\text{five}} + 24_{\text{five}} =$ ________ five. 41

131. Now try these:

$47_{\text{nine}} + 3_{\text{nine}} = 51_{\text{nine}}$

$47_{\text{nine}} + 13_{\text{nine}} =$ ________ nine 61

$47_{\text{nine}} + 23_{\text{nine}} =$ ________ nine 71

132. In base seven:

$34_{\text{seven}} + 6_{\text{seven}} =$ ________ seven 43

$34_{\text{seven}} + 16_{\text{seven}} =$ ________ seven 53

$34_{\text{seven}} + 26_{\text{seven}} =$ ________ seven 63

133. $42_{\text{eight}} + 27_{\text{eight}} =$ ______ eight. 71

134. $25_{\text{six}} + 25_{\text{six}} =$ ______ six. 54

135. Sometimes the addends are written one above the other. Try this one.

$$\begin{array}{r} 14_{\text{five}} \\ +\,24_{\text{five}} \\ \hline \end{array}$$

______ five 43

136.

$$\begin{array}{r} 146_{\text{seven}} \\ +13_{\text{seven}} \\ \hline \end{array}$$

______ seven 162

137.

$$\begin{array}{r} 104_{\text{five}} \\ +24_{\text{five}} \\ \hline \end{array}$$

______ five 133

138. Find the sum in each of the following cases:

$$\begin{array}{r} 33_{\text{five}} \\ +12_{\text{five}} \\ \hline \end{array}$$

______ five 100

$$\begin{array}{r} 22_{\text{four}} \\ +13_{\text{four}} \\ \hline \end{array}$$

______ four 101

$$\begin{array}{r} 125_{\text{six}} \\ +42_{\text{six}} \\ \hline \end{array}$$

______ six 211

$$\begin{array}{r} 105_{\text{six}} \\ +235_{\text{six}} \\ \hline \end{array}$$

______ six 344

TRY YOUR HAND

Here are some exercises which will help you to review and extend the ideas you have seen in Chapter 2. The answers to these problems appear on page 168.

1. Try these addition problems in various bases:

   a. $314_{\text{five}} + 211_{\text{five}}$
   b. $274_{\text{eight}} + 1066_{\text{eight}}$
   c. $555_{\text{six}} + 4_{\text{six}}$
   d. $42_{\text{six}} + 31_{\text{six}} + 20_{\text{six}}$
   e. $11_{\text{three}} + 21_{\text{three}} + 12_{\text{three}}$
   f. $201_{\text{four}} + 22_{\text{four}} + 10_{\text{four}}$
   g. $18_{\text{nine}} + 15_{\text{nine}} + 12_{\text{nine}}$
   h. $30_{\text{eight}} + 50_{\text{eight}} + 70_{\text{eight}}$
   i. $121_{\text{three}} + 201_{\text{three}} + 211_{\text{three}}$
   j. $44_{\text{five}} + 100_{\text{five}} + 1311_{\text{five}} + 2_{\text{five}}$

2. Here are some addition problems in base three:

   a. $$\begin{array}{r} 211_{\text{three}} \\ +222_{\text{three}} \\ \hline \end{array}$$

   b. $$\begin{array}{r} 112_{\text{three}} \\ +2_{\text{three}} \\ \hline \end{array}$$

   c. $$\begin{array}{r} 122_{\text{three}} \\ +21_{\text{three}} \\ \hline \end{array}$$

   d. $$\begin{array}{r} 1012_{\text{three}} \\ +212_{\text{three}} \\ \hline \end{array}$$

3. Here are some addition problems in base ten. Use the methods described in this chapter to find the answer to each.

   a. $15_{\text{ten}} + 9_{\text{ten}}$
   b. $119_{\text{ten}} + 8_{\text{ten}}$
   c. $201_{\text{ten}} + 10_{\text{ten}}$
   d. $397_{\text{ten}} + 8_{\text{ten}}$
   e. $47_{\text{ten}} + 16_{\text{ten}}$
   f. $238_{\text{ten}} + 62_{\text{ten}}$
   g. $900_{\text{ten}} + 2400_{\text{ten}}$
   h. $200_{\text{ten}} + 30_{\text{ten}} + 5_{\text{ten}}$
   i. $604_{\text{ten}} + 10_{\text{ten}} + 7_{\text{ten}}$
   j. $333_{\text{ten}} + 67_{\text{ten}} + 10_{\text{ten}}$

4. Here are some addition problems already completed for you. Determine the base in which each problem was done.

a. $4_? + 3_? = 10_?$
b. $6_? + 3_? = 11_?$
c. $22_? + 14_? = 41_?$
d. $200_? + 300_? = 1100_?$
e. $2_? + 4_? + 3_? = 12_?$
f. $2_? + 3_? + 3_? = 20_?$
*g. $8_? + 3_? = 10_?$
*h. $9_? + 7_? = 12_?$
*i. $1_? + 1_? = 10_?$
*j. $9_? + 19_? = 21_?$

5. Find the missing addend in each of the following cases.

a. $4_{\text{seven}} + ?_{\text{seven}} = 11_{\text{seven}}$
b. $9_{\text{ten}} + ?_{\text{ten}} = 16_{\text{ten}}$
c. $18_{\text{nine}} + ?_{\text{nine}} = 24_{\text{nine}}$
d. $26_{\text{seven}} + ?_{\text{seven}} = 40_{\text{seven}}$
e. $31_{\text{six}} + ?_{\text{six}} = 50_{\text{six}}$
f. $42_{\text{five}} + ?_{\text{five}} = 103_{\text{five}}$
g. $32_{\text{four}} + ?_{\text{four}} = 110_{\text{four}}$
*h. $16_{\text{twelve}} + ?_{\text{twelve}} = 20_{\text{twelve}}$
*i. $27_{\text{fifteen}} + ?_{\text{fifteen}} = 30_{\text{fifteen}}$
*j. $16_{\text{twenty}} + ?_{\text{twenty}} = 26_{\text{twenty}}$

6. Here are some questions in which you are asked to add two numbers represented in *different* bases. You'll have to rewrite each question so that the numerals are in the same base. In each case, give the answer in the larger base.

a. $4_{\text{six}} + 5_{\text{seven}}$
b. $3_{\text{eight}} + 4_{\text{five}}$
c. $3_{\text{five}} + 6_{\text{seven}}$
d. $10_{\text{seven}} + 2_{\text{three}}$
e. $11_{\text{five}} + 8_{\text{nine}}$
f. $21_{\text{nine}} + 7_{\text{eight}}$
g. $33_{\text{four}} + 10_{\text{eight}}$
h. $32_{\text{six}} + 12_{\text{three}}$
i. $29_{\text{ten}} + 16_{\text{seven}}$
j. $40_{\text{six}} + 50_{\text{seven}}$

7. One way to check addition problems in various bases (and in base ten, too) is to add the numbers in each addend until you have "reduced" the numbers to a one-digit number. Then you add these numbers. This sum should equal the "reduced" answer.

For example:

$$\begin{array}{rcl} 21_{\text{five}} & \rightarrow 2_{\text{five}} + 1_{\text{five}} = & 3_{\text{five}} \\ \underline{+10_{\text{five}}} & \rightarrow 1_{\text{five}} + 0_{\text{five}} = & \underline{+1_{\text{five}}} \\ 31_{\text{five}} & \rightarrow 3_{\text{five}} + 1_{\text{five}} = & 4_{\text{five}} \end{array}$$

Use the check to see whether each of the following answers is correct:

a. $11_{\text{three}} + 2_{\text{three}} = 20_{\text{three}}$
b. $40_{\text{seven}} + 21_{\text{seven}} = 62_{\text{seven}}$
c. $53_{\text{ten}} + 28_{\text{ten}} = 81_{\text{ten}}$
d. $33_{\text{five}} + 4_{\text{five}} = 42_{\text{five}}$
e. $41_{\text{six}} + 23_{\text{six}} = 103_{\text{six}}$

*8. Here are some addition problems in base eleven. Find the answer to each.

a. $4_{\text{eleven}} + 3_{\text{eleven}}$

b. $31_{\text{eleven}} + 35_{\text{eleven}}$

c. $8_{\text{eleven}} + 3_{\text{eleven}}$

d. $18_{\text{eleven}} + 5_{\text{eleven}}$

e. $300_{\text{eleven}} + 1500_{\text{eleven}}$

f. $700_{\text{eleven}} + 2700_{\text{eleven}}$

g. $1300_{\text{eleven}} + 40_{\text{eleven}} + 7_{\text{eleven}}$

h. $200_{\text{eleven}} + 16_{\text{eleven}} + 8_{\text{eleven}}$

i. $60_{\text{eleven}} + 20_{\text{eleven}} + 70_{\text{eleven}}$

j. $15_{\text{eleven}} + 81_{\text{eleven}} + 39_{\text{eleven}}$

*9. Here are some addition problems in bases *greater than ten.* Find the answer to each.

a. $4_{\text{eleven}} + 5_{\text{eleven}}$

b. $13_{\text{twelve}} + 4_{\text{twelve}}$

c. $18_{\text{sixteen}} + 8_{\text{sixteen}}$

d. $20_{\text{fifteen}} + 30_{\text{fifteen}}$

e. $14_{\text{twelve}} + 9_{\text{twelve}}$

f. $33_{\text{twenty}} + 35_{\text{twenty}}$

g. $48_{\text{fifteen}} + 8_{\text{fifteen}}$

h. $66_{\text{twelve}} + 8_{\text{twelve}}$

i. $480_{\text{fifteen}} + 90_{\text{fifteen}}$

j. $20_{\text{fifty}} + 30_{\text{fifty}}$

THINK IT OVER

Here is another chance to look into the "why" of what you learned in Chapter 2. Don't be surprised if you come up with several possible answers to a particular question.

1. Sometimes the answer to an addition problem is the same as it would be in base ten. For example, $21_{\text{five}} + 13_{\text{five}} = 34_{\text{five}}$ and sometimes the answer is very much different from what it would be in base ten. For example, $33_{\text{five}} + 4_{\text{five}} = 42_{\text{five}}$. Why is this so?

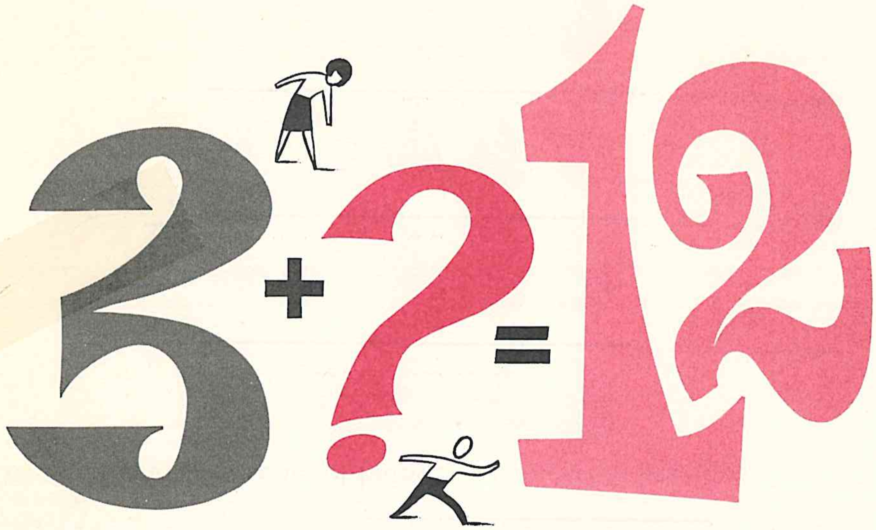
2. When you first learned how to add, you learned how to "carry" numbers. How is "carrying" related to the way addition is done in this chapter?

3. Can you add two numbers such as 23_{eight} and 41_{seven} without changing the numerals to the same base? Why?

4. How can you use what you learned in this chapter to find the total time for a job if one part takes two weeks and one day, and the second part takes three weeks and six days? How can you use addition in other bases to help you solve everyday problems?

*5. Look at example 7 of the Try Your Hand section of this chapter. Is this a foolproof method of checking? Why?

3 A Look at Subtraction



A PROBLEM TO SOLVE

Suppose someone asks you to solve the base five problem $3_{\text{five}} + ?_{\text{five}} = 12_{\text{five}}$. Would you be able to find out what number the question mark represents? How would you go about it? Would you add? Would you subtract? Would you say that the question mark stands for 4?

In this chapter you are going to learn how to **subtract** in various bases. The subtraction methods we shall use are probably very different from the methods you've seen before. You are invited to compare the "new" methods with the methods you've been using in your regular work.

Subtraction and Addition

1. Let's look at some "old" addition facts in base five and then use them to discover "new" facts about subtraction in base five.

   This frame and the next few frames consist of two parts. The first part deals with addition in base five—you learned how to add in Chapter 2. The second part deals with subtraction—here you'll have to take a *guess*, because you don't yet know how to subtract in base five.

   $1_{\text{five}} + \underline{\quad\quad}_{\text{five}} = 4_{\text{five}}.$ 3

   $4_{\text{five}} - 1_{\text{five}} = \underset{guess}{\underline{\quad\quad}}_{\text{five}}.$ 3

2. $1_{five} + \_\_\_\__{five} = 3_{five}$. 2

$3_{five} - 1_{five} = \underset{guess}{\_\_\_\_}_{five}$. 2

3. $2_{five} + \_\_\_\__{five} = 4_{five}$. 2

$4_{five} - 2_{five} = \underset{guess}{\_\_\_\_}_{five}$. 2

4. $3_{five} + \_\_\_\__{five} = 4_{five}$. 1

$4_{five} - 3_{five} = \underset{guess}{\_\_\_\_}_{five}$. 1

5. $1_{five} + \_\_\_\__{five} = 12_{five}$. 11

$12_{five} - 1_{five} = \underset{guess}{\_\_\_\_}_{five}$. 11

6. $3_{five} + \_\_\_\__{five} = 14_{five}$. 11

$14_{five} - 3_{five} = \underset{guess}{\_\_\_\_}_{five}$. 11

7. $2_{five} + \_\_\_\__{five} = 12_{five}$. 10

$12_{five} - 2_{five} = \underset{guess}{\_\_\_\_}_{five}$. 10

8. Were your guesses correct? Do you see how addition and subtraction are related?

Instead of writing the addition problem $4_{five} + ?_{five} = 14_{five}$, you could write the subtraction problem ——?——.

a. $14_{five} - 4_{five} = ?_{five}$
b. $4_{five} - 14_{five} = ?_{five}$
c. $?_{five} - 14_{five} = 4_{five}$

a. $14_{five} - 4_{five} = ?_{five}$

9. If you were asked to find an answer to $14_{five} - 1_{five} = ?_{five}$, you could rewrite the problem as ——?——.

a. $14_{five} + ?_{five} = 1_{five}$
b. $14_{five} - ?_{five} = 1_{five}$
c. $1_{five} + ?_{five} = 14_{five}$

c. $1_{five} + ?_{five} = 14_{five}$

10. The subtraction problem $13_{\text{five}} - 2_{\text{five}} = ?_{\text{five}}$ is the same as the addition problem ——?——.

a. $2_{\text{five}} + 13_{\text{five}} = ?_{\text{five}}$

b. $2_{\text{five}} + ?_{\text{five}} = 13_{\text{five}}$ — **b.** $2_{\text{five}} + ?_{\text{five}} = 13_{\text{five}}$

c. $13_{\text{five}} + ?_{\text{five}} = 2_{\text{five}}$

11. Fill in the missing addends in base five:

1. $4_{\text{five}} + \_\_\_\__{\text{five}} = 4_{\text{five}}$ — 0
2. $3_{\text{five}} + \_\_\_\__{\text{five}} = 4_{\text{five}}$ — 1
3. $1_{\text{five}} + \_\_\_\__{\text{five}} = 4_{\text{five}}$ — 3
4. $2_{\text{five}} + \_\_\_\__{\text{five}} = 3_{\text{five}}$ — 1
5. $1_{\text{five}} + \_\_\_\__{\text{five}} = 3_{\text{five}}$ — 2

12. Now try the following subtraction problems:

1. $4_{\text{five}} - 1_{\text{five}} = \_\_\_\__{\text{five}}$ — 3
2. $2_{\text{five}} - 2_{\text{five}} = \_\_\_\__{\text{five}}$ — 0
3. $4_{\text{five}} - 3_{\text{five}} = \_\_\_\__{\text{five}}$ — 1
4. $3_{\text{five}} - 1_{\text{five}} = \_\_\_\__{\text{five}}$ — 2
5. $2_{\text{five}} - 1_{\text{five}} = \_\_\_\__{\text{five}}$ — 1

13. In base nine, find the missing addends:

1. $6_{\text{nine}} + \_\_\_\__{\text{nine}} = 8_{\text{nine}}$ — 2
2. $4_{\text{nine}} + \_\_\_\__{\text{nine}} = 5_{\text{nine}}$ — 1
3. $2_{\text{nine}} + \_\_\_\__{\text{nine}} = 8_{\text{nine}}$ — 6
4. $0_{\text{nine}} + \_\_\_\__{\text{nine}} = 6_{\text{nine}}$ — 6
5. $4_{\text{nine}} + \_\_\_\__{\text{nine}} = 7_{\text{nine}}$ — 3

14. And in base nine do the following subtraction examples:

1. $8_{\text{nine}} - 5_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 3
2. $6_{\text{nine}} - 3_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 3
3. $7_{\text{nine}} - 2_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 5
4. $8_{\text{nine}} - 4_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 4
5. $8_{\text{nine}} - 8_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 0

15. Try the following subtraction examples:

1. $6_{\text{seven}} - 5_{\text{seven}} = \_\_\_\__{\text{seven}}$ 1
2. $2_{\text{three}} - 2_{\text{three}} = \_\_\_\__{\text{three}}$ 0
3. $8_{\text{nine}} - 4_{\text{nine}} = \_\_\_\__{\text{nine}}$ 4
4. $7_{\text{eight}} - 1_{\text{eight}} = \_\_\_\__{\text{eight}}$ 6
5. $4_{\text{six}} - 2_{\text{six}} = \_\_\_\__{\text{six}}$ 2

16. In base seven, you know that

$$\mathbf{5}_{\text{seven}} - \mathbf{3}_{\text{seven}} = \mathbf{2}_{\text{seven}}.$$

Then it is easy to see that

$$\mathbf{50}_{\text{seven}} - \mathbf{30}_{\text{seven}} = \text{——?——}.$$

a. 2_{seven} **b.** 20_{seven} **c.** 200_{seven} — **b.** 20_{seven}

17. In base nine,

$$\mathbf{8}_{\text{nine}} - \mathbf{3}_{\text{nine}} = \mathbf{5}_{\text{nine}},$$

and thus

$$\mathbf{800}_{\text{nine}} - \mathbf{300}_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}.$$ 500

18. Try these subtraction problems:

1. $40_{\text{five}} - 20_{\text{five}} = \_\_\_\__{\text{five}}$ 20
2. $300_{\text{five}} - 200_{\text{five}} = \_\_\_\__{\text{five}}$ 100
3. $2000_{\text{five}} - 1000_{\text{five}} = \_\_\_\__{\text{five}}$ 1000
4. $4000_{\text{five}} - 3000_{\text{five}} = \_\_\_\__{\text{five}}$ 1000
5. $4000_{\text{five}} - 2000_{\text{five}} = \_\_\_\__{\text{five}}$ 2000

19. Here are some subtraction problems in different bases:

1. $7000_{\text{nine}} - 2000_{\text{nine}} = \_\_\_\__{\text{nine}}$ 5000
2. $600_{\text{eight}} - 500_{\text{eight}} = \_\_\_\__{\text{eight}}$ 100
3. $20_{\text{seven}} - 10_{\text{seven}} = \_\_\_\__{\text{seven}}$ 10
4. $300_{\text{four}} - 200_{\text{four}} = \_\_\_\__{\text{four}}$ 100
5. $2000_{\text{three}} - 2000_{\text{three}} = \_\_\_\__{\text{three}}$ 0

20. In base seven,

$$3_{\text{seven}} - 1_{\text{seven}} = 2_{\text{seven}},$$

and so

$$23_{\text{seven}} - 21_{\text{seven}} = \text{—?—}.$$

a. 0_{seven} b. 12_{seven} c. 2_{seven}

c. 2_{seven}

21. $67_{\text{eight}} - 62_{\text{eight}} =$ —?—.

a. 5_{eight} b. 15_{eight} c. 25_{eight}

a. 5_{eight}

22. Try these subtraction problems:

1. $38_{\text{nine}} - 31_{\text{nine}} =$ ——— nine 7
2. $48_{\text{nine}} - 41_{\text{nine}} =$ ——— nine 7
3. $44_{\text{seven}} - 42_{\text{seven}} =$ ——— seven 2
4. $44_{\text{five}} - 42_{\text{five}} =$ ——— five 2
5. $37_{\text{eight}} - 32_{\text{eight}} =$ ——— eight 5

23. Of course, it isn't difficult to see that

$134_{\text{seven}} - 131_{\text{seven}}$ is ——— seven. 3

24. $812_{\text{nine}} - 810_{\text{nine}} =$ ——— nine. 2

25. $812_{\text{nine}} - 800_{\text{nine}} =$ —?—.

a. 2_{nine} b. 12_{nine} c. 0_{nine}

b. 12_{nine}

26. $644_{\text{seven}} - 600_{\text{seven}} =$ ——— seven. 44

27. $4132_{\text{five}} - 4000_{\text{five}} =$ ——— five. 132

28. $4224_{five} - 4100_{five} =$ ______ $_{five}$. 124

29. In base seven, $355_{seven} - 341_{seven} =$ ______ $_{seven}$. 14

30. Try each of these problems:

1. $342_{five} - 311_{five} =$ ______ $_{five}$ 31
2. $42_{five} - 31_{five} =$ ______ $_{five}$ 11
3. $33_{five} - 21_{five} =$ ______ $_{five}$ 12
4. $434_{five} - 210_{five} =$ ______ $_{five}$ 224
5. $342_{five} - 101_{five} =$ ______ $_{five}$ 241

31. Here are some subtraction problems in base nine:

1. $84_{nine} - 72_{nine} =$ ______ $_{nine}$ 12
2. $66_{nine} - 32_{nine} =$ ______ $_{nine}$ 34
3. $176_{nine} - 44_{nine} =$ ______ $_{nine}$ 132
4. $4085_{nine} - 23_{nine} =$ ______ $_{nine}$ 4062
5. $10077_{nine} - 56_{nine} =$ ______ $_{nine}$ 10021

32. Find the answer to each of the following subtraction problems:

1. $14_{five} - 10_{five} =$ ______ $_{five}$ 4
2. $42_{six} - 11_{six} =$ ______ $_{six}$ 31
3. $77_{nine} - 66_{nine} =$ ______ $_{nine}$ 11
4. $275_{eight} - 263_{eight} =$ ______ $_{eight}$ 12
5. $444_{five} - 231_{five} =$ ______ $_{five}$ 213

33. For practice, find the answer to each of these:

1. $43_{\text{five}} - 20_{\text{five}} = \_\_\_\_\_\__{\text{five}}$ 23

2. $74_{\text{eight}} - 40_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}$ 34

3. $125_{\text{seven}} - 20_{\text{seven}} = \_\_\_\_\_\__{\text{seven}}$ 105

4. $88_{\text{nine}} - 10_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}$ 78

5. $97_{\text{ten}} - 30_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}$ 67

Another Look at Subtraction

34. We haven't told the *full* story of subtraction yet. We haven't dealt with problems such as $43_{\text{seven}} - 26_{\text{seven}}$ and $32_{\text{five}} - 14_{\text{five}}$. To deal with such problems, we're going to use a "new" method—a method which is quite simple because it depends upon addition.

Sometimes two subtraction problems have the same answer, even though the problems don't look alike. For example,

$4_{\text{five}} - 2_{\text{five}} = \_\_\_\__{\text{five}}$ and $3_{\text{five}} - 1_{\text{five}} = \_\_\_\__{\text{five}}$. 2, 2

35. In base ten, $48_{\text{ten}} - 6_{\text{ten}} = 52_{\text{ten}} - \_\_\_\_\_\__{\text{ten}}$. 10

36. In base six, $12_{\text{six}} - 1_{\text{six}} = 15_{\text{six}} - \_\_\_\__{\text{six}}$. 4

37. $34_{\text{seven}} - 32_{\text{seven}} = \_\_?\_\_ - 44_{\text{seven}}$.

a. 51_{seven} b. 46_{seven} c. 35_{seven} **b.** 46_{seven}

38. $42_{\text{five}} - 20_{\text{five}} = \_\_\_\_\_\__{\text{five}} - 22_{\text{five}}$. 44

39. $13_{\text{eight}} - 11_{\text{eight}} = 17_{\text{eight}} - \_\_\_\_\_\__{\text{eight}}$. 15

40. A problem which has the same answer as $26_{nine} - 21_{nine}$ is ——?——.

a. $28_{nine} - 23_{nine}$ **b.** $27_{nine} - 20_{nine}$ **c.** $30_{nine} - 21_{nine}$ **a.** $28_{nine} - 23_{nine}$

41. In base seven, $13_{seven} - 1_{seven} =$ ______ $_{seven}$. 12

Suppose we add 3_{seven} to both 13_{seven} and 1_{seven}. The new problem, then, is $16_{seven} - 4_{seven}$. And $16_{seven} - 4_{seven} =$ ______ $_{seven}$. 12

In other words, adding 3_{seven} ——?—— change the answer to the original subtraction problem, $13_{seven} - 1_{seven}$.

a. does **b.** does not **b.** does not

42. In base five, $24_{five} - 22_{five} =$ ______ $_{five}$. 2

Suppose we add 10_{five} to both 24_{five} and 22_{five}. Then our new problem is $34_{five} - 32_{five}$, and $34_{five} - 32_{five} =$ ______ $_{five}$. 2

The answers to the two subtraction problems are ——?——.

a. different **b.** the same **b.** the same

43. In base ten, $35_{ten} - 30_{ten} =$ ______ $_{ten}$. 5

Suppose we add 4_{ten} to both 35_{ten} and 30_{ten}. Then our new subtraction problem is $39_{ten} - 34_{ten}$. And $39_{ten} - 34_{ten} =$ ______ $_{ten}$. 5

In other words, the answer ——?—— change.

a. does **b.** does not **b.** does not

44. Now suppose we are asked to do the problem $10_{five} - 4_{five}$. By adding the same number to both 10_{five} and 4_{five}, we can find another subtraction problem which has the same answer.

Suppose we add 1_{five} to both numbers. Then our new problem is ——?——.

a. $11_{five} - 10_{five}$ **b.** $10_{five} - 11_{five}$ **c.** $11_{five} - 4_{five}$ **a.** $11_{five} - 10_{five}$

45. Since the answer to $11_{five} - 10_{five}$ is ______ $_{five}$, the answer to 1

$10_{five} - 4_{five}$ is also ______ $_{five}$. 1

46. One way to find the answer to $32_{five} - 14_{five}$ is to transform the problem by adding the same number to both 32_{five} and 14_{five}. This method of subtraction is called the **equal additions** method. Instead of writing $32_{five} - 14_{five}$, you can add 1_{five} to both 32_{five} and 14_{five}. The new problem is $\_\_\_\_\__{five} - 20_{five}$. 33

You know that the answers to the two problems will be ——?——.

a. different **b.** the same **b.** the same

47. Instead of writing $47_{nine} - 28_{nine}$, you can write another problem: $\_\_\_\_\__{nine} - 30_{nine}$. 48

48. Another way to write the problem $41_{five} - 24_{five}$ is to write $\_\_\_\_\__{five} - 30_{five}$. 42

49. Fill in the blanks below to transform each subtraction problem:

1. $21_{four} - 12_{four} = 23_{four} - \_\_\_\_\__{four}$ 20
2. $34_{seven} - 15_{seven} = 36_{seven} - \_\_\_\_\__{seven}$ 20
3. $100_{five} - 32_{five} = 103_{five} - \_\_\_\_\__{five}$ 40
4. $63_{eight} - 44_{eight} = 67_{eight} - \_\_\_\_\__{eight}$ 50
5. $241_{five} - 222_{five} = 244_{five} - \_\_\_\_\__{five}$ 230

50. If one number is to be subtracted from another, and if you add a number to *both* numbers, the answer to the subtraction problem ——?——.

a. changes **b.** remains the same **b.** remains the same

51. Instead of writing $43_{ten} - 27_{ten}$, you could write $\_\_\_\_\__{ten} - 30_{ten}$. 46

The answer to this new problem is $\_\_\_\_\__{ten}$, and thus the answer 16

to $43_{ten} - 27_{ten}$ is $\_\_\_\_\__{ten}$. 16

52. Instead of writing $20_{five} - 2_{five}$, you could write $23_{five} - \_\_\_\_\__{five}$. 10

The answer to both problems is $\_\_\_\_\__{five}$. 13

53. $71_{\text{eight}} - 57_{\text{eight}} =$ ______ $_{\text{eight}} - 60_{\text{eight}}$. And 72

$72_{\text{eight}} - 60_{\text{eight}} =$ ______ $_{\text{eight}}$. So the answer to the original prob- 12

lem, $71_{\text{eight}} - 57_{\text{eight}}$, is ______ $_{\text{eight}}$. 12

54. Here are some subtraction problems. Use the equal additions method to find the answer to each one.

1. $42_{\text{five}} - 24_{\text{five}} =$ ______ $_{\text{five}} - 30_{\text{five}} =$ ______ $_{\text{five}}$ 43, 13
2. $67_{\text{ten}} - 48_{\text{ten}} =$ ______ $_{\text{ten}} - 50_{\text{ten}} =$ ______ $_{\text{ten}}$ 69, 19
3. $60_{\text{seven}} - 33_{\text{seven}} =$ ______ $_{\text{seven}} - 40_{\text{seven}} =$ ______ $_{\text{seven}}$ 64, 24
4. $32_{\text{eight}} - 7_{\text{eight}} =$ ______ $_{\text{eight}} - 10_{\text{eight}} =$ ______ $_{\text{eight}}$ 33, 23
5. $76_{\text{nine}} - 47_{\text{nine}} =$ ______ $_{\text{nine}} - 50_{\text{nine}} =$ ______ $_{\text{nine}}$ 78, 28

55. Now try these subtraction problems:

1. $50_{\text{seven}} - 26_{\text{seven}} =$ ______ $_{\text{seven}} - 30_{\text{seven}} =$ ______ $_{\text{seven}}$ 51, 21
2. $42_{\text{five}} - 13_{\text{five}} =$ ______ $_{\text{five}} - 20_{\text{five}} =$ ______ $_{\text{five}}$ 44, 24
3. $84_{\text{ten}} - 58_{\text{ten}} =$ ______ $_{\text{ten}} - 60_{\text{ten}} =$ ______ $_{\text{ten}}$ 86, 26
4. $21_{\text{three}} - 12_{\text{three}} =$ ______ $_{\text{three}} - 20_{\text{three}} =$ ______ $_{\text{three}}$ 22, 2
5. $71_{\text{eight}} - 56_{\text{eight}} =$ ______ $_{\text{eight}} - 60_{\text{eight}} =$ ______ $_{\text{eight}}$ 73, 13

56. Use the equal additions method to find the answers to the problems below.

1. $45_{\text{eight}} - 27_{\text{eight}} =$ ______ $_{\text{eight}}$ 16
2. $32_{\text{five}} - 14_{\text{five}} =$ ______ $_{\text{five}}$ 13
3. $62_{\text{ten}} - 47_{\text{ten}} =$ ______ $_{\text{ten}}$ 15
4. $32_{\text{four}} - 13_{\text{four}} =$ ______ $_{\text{four}}$ 13
5. $64_{\text{nine}} - 36_{\text{nine}} =$ ______ $_{\text{nine}}$ 27

120 − 40 = ??

57. To solve the problem $120_{\text{five}} - 40_{\text{five}}$, we could add 10_{five} to both 120_{five} and 40_{five}. The new problem, then, is ——?——.

a. $130_{\text{five}} - 40_{\text{five}}$ **b.** $120_{\text{five}} - 100_{\text{five}}$ **c.** $130_{\text{five}} - 100_{\text{five}}$

c. $130_{\text{five}} - 100_{\text{five}}$

58. To solve the problem $420_{\text{six}} - 230_{\text{six}}$, we find the answer to

$\_\_\_\_\_\__{\text{six}} - 300_{\text{six}}$.

450

59. Instead of working with $840_{\text{nine}} - 370_{\text{nine}}$, we could find the answer to $\_\_\_\_\_\__{\text{nine}} - 400_{\text{nine}}$.

860

60. Try using the method of equal additions on these problems:

1. $140_{\text{eight}} - 70_{\text{eight}} = \_\_\_\__{\text{eight}} - 100_{\text{eight}} = \_\_\_\__{\text{eight}}$ — 150, 50
2. $320_{\text{ten}} - 60_{\text{ten}} = \_\_\_\__{\text{ten}} - 100_{\text{ten}} = \_\_\_\__{\text{ten}}$ — 360, 260
3. $410_{\text{seven}} - 260_{\text{seven}} = \_\_\_\__{\text{seven}} - 300_{\text{seven}} = \_\_\_\__{\text{seven}}$ — 420, 120
4. $1600_{\text{nine}} - 800_{\text{nine}} = \_\_\_\__{\text{nine}} - 1000_{\text{nine}} = \_\_\_\__{\text{nine}}$ — 1700, 700
5. $320_{\text{four}} - 130_{\text{four}} = \_\_\_\__{\text{four}} - 200_{\text{four}} = \_\_\_\__{\text{four}}$ — 330, 130

61. And now try these:

1. $320_{\text{five}} - 40_{\text{five}} = \_\_\_\_\_\__{\text{five}}$ — 230
2. $1400_{\text{nine}} - 800_{\text{nine}} = \_\_\_\_\_\__{\text{nine}}$ — 500
3. $210_{\text{ten}} - 80_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}$ — 130
4. $1200_{\text{four}} - 300_{\text{four}} = \_\_\_\_\_\__{\text{four}}$ — 300
5. $460_{\text{eight}} - 270_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}$ — 170

62. Subtraction problems are often written like this:

$$\begin{array}{r} 1300_{\text{five}} \\ -400_{\text{five}} \\ \hline \end{array}$$

For the rest of this chapter we will use this form to write subtraction problems. To find the answer to the problem above, we can use the equal additions method:

$$\begin{array}{r} 1300_{\text{five}} \\ -400_{\text{five}} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} \underline{\qquad}_{\text{five}} \\ -1000_{\text{five}} \\ \hline \end{array}$$ 1400

The answer to the original problem is ________ five. 400

63. Sometimes subtraction problems are complicated and we need to make *two* equal additions.

For example, suppose we wanted to find the answer to:

$$\begin{array}{r} 312_{\text{five}} \\ -23_{\text{five}} \\ \hline \end{array}$$

We could add 2_{five} to both 312_{five} and 23_{five}. Then we have:

$$\begin{array}{r} 314_{\text{five}} \\ -30_{\text{five}} \\ \hline \end{array}$$

Then we could add 20_{five} to both 314_{five} and 30_{five} to get:

$$\begin{array}{r} 334_{\text{five}} \\ -100_{\text{five}} \\ \hline \end{array}$$

The answer to this problem is ________ five and thus we have obtained an answer to the original problem. 234

64. Suppose we try the following problem:

$$\begin{array}{r} 154_{\text{eight}} \\ -66_{\text{eight}} \\ \hline \end{array}$$

First we can add 2_{eight} to both 154_{eight} and 66_{eight}, obtaining:

$$\begin{array}{r} 156_{\text{eight}} \\ -70_{\text{eight}} \\ \hline \end{array}$$

Then we can add 10_{eight} to both 156_{eight} and 70_{eight} to get:

$$\begin{array}{r} 166_{\text{eight}} \\ -100_{\text{eight}} \\ \hline \end{array}$$

The answer to the original problem is ______ eight. 66

65. Of course, you can extend this method to deal with problems which are quite difficult.

For example, here are the steps involved in finding the answer to $1122_{\text{five}} - 324_{\text{five}}$:

1123, 1143, 1243

$$\begin{array}{r} 1122_{\text{five}} \\ -324_{\text{five}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \underline{\qquad}_{\text{five}} \\ -330_{\text{five}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \underline{\qquad}_{\text{five}} \\ -400_{\text{five}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \underline{\qquad}_{\text{five}} \\ -1000_{\text{five}} \\ \hline \end{array}$$

The answer to the original problem, $1122_{\text{five}} - 324_{\text{five}}$, is

________ five.

243

66. Try each of the following problems:

$$\begin{array}{r} 101_{\text{five}} \\ -23_{\text{five}} \\ \hline \end{array}$$

______ five

23

$$\begin{array}{r} 1030_{\text{eight}} \\ -470_{\text{eight}} \\ \hline \end{array}$$

______ eight

340

$$\begin{array}{r} 1231_{\text{four}} \\ -313_{\text{four}} \\ \hline \end{array}$$

______ four

312

$$\begin{array}{r} 1006_{\text{eight}} \\ -227_{\text{eight}} \\ \hline \end{array}$$

______ eight

557

$$\begin{array}{r} 47309_{\text{ten}} \\ -18415_{\text{ten}} \\ \hline \end{array}$$

______ ten

28894

$$\begin{array}{r} 75402_{\text{ten}} \\ -14511_{\text{ten}} \\ \hline \end{array}$$

______ ten

60891

Try Your Hand

■ The questions in this section will give you a chance to review and extend what you learned in this chapter. The answers to these questions appear on page 169.

1. Find answers to the following subtraction problems.

   a. $8_{\text{nine}} - 5_{\text{nine}}$
   b. $23_{\text{four}} - 11_{\text{four}}$
   c. $217_{\text{eight}} - 104_{\text{eight}}$
   d. $406_{\text{seven}} - 301_{\text{seven}}$
   e. $5200_{\text{six}} - 2000_{\text{six}}$
   f. $2104_{\text{five}} - 1102_{\text{five}}$
   g. $8974_{\text{ten}} - 5321_{\text{ten}}$
   h. $1175_{\text{nine}} - 150_{\text{nine}}$
   i. $3020_{\text{four}} - 20_{\text{four}}$
   j. $2756_{\text{eight}} - 1111_{\text{eight}}$

2. Try these subtraction problems.

   a. $21_{\text{three}} - 12_{\text{three}}$
   b. $21_{\text{four}} - 12_{\text{four}}$
   c. $21_{\text{five}} - 12_{\text{five}}$
   d. $21_{\text{six}} - 12_{\text{six}}$
   e. $21_{\text{seven}} - 12_{\text{seven}}$
   f. $21_{\text{eight}} - 12_{\text{eight}}$
   g. $21_{\text{nine}} - 12_{\text{nine}}$
   h. $21_{\text{ten}} - 12_{\text{ten}}$

3. All these problems are in base nine. Find the answer to each one.

   a. $26_{\text{nine}} - 26_{\text{nine}}$
   b. $26_{\text{nine}} - 25_{\text{nine}}$
   c. $26_{\text{nine}} - 24_{\text{nine}}$
   d. $26_{\text{nine}} - 23_{\text{nine}}$
   e. $26_{\text{nine}} - 22_{\text{nine}}$
   f. $26_{\text{nine}} - 21_{\text{nine}}$
   g. $26_{\text{nine}} - 20_{\text{nine}}$
   h. $26_{\text{nine}} - 18_{\text{nine}}$
   i. $26_{\text{nine}} - 17_{\text{nine}}$
   j. $26_{\text{nine}} - 16_{\text{nine}}$

4. These problems are in base five. Find the answer to each one.

a. $4_{\text{five}} - 3_{\text{five}}$

b. $40_{\text{five}} - 30_{\text{five}}$

c. $400_{\text{five}} - 300_{\text{five}}$

d. $34_{\text{five}} - 33_{\text{five}}$

e. $340_{\text{five}} - 330_{\text{five}}$

f. $44_{\text{five}} - 33_{\text{five}}$

g. $444_{\text{five}} - 333_{\text{five}}$

h. $400_{\text{five}} - 30_{\text{five}}$

i. $400_{\text{five}} - 330_{\text{five}}$

j. $400_{\text{five}} - 33_{\text{five}}$

5. Use the method of equal additions, where necessary, to find answers to the following problems.

a. $39_{\text{ten}} - 26_{\text{ten}}$

b. $33_{\text{seven}} - 6_{\text{seven}}$

c. $81_{\text{nine}} - 26_{\text{nine}}$

d. $108_{\text{nine}} - 18_{\text{nine}}$

e. $251_{\text{six}} - 113_{\text{six}}$

f. $751_{\text{eight}} - 625_{\text{eight}}$

g. $204_{\text{ten}} - 116_{\text{ten}}$

h. $115_{\text{seven}} - 106_{\text{seven}}$

i. $213_{\text{four}} - 132_{\text{four}}$

j. $1040_{\text{five}} - 41_{\text{five}}$

6. Each of the following problems is in base ten. Use the equal additions method, where necessary, to find the answer to each.

a. $24_{\text{ten}} - 16_{\text{ten}}$

b. $39_{\text{ten}} - 15_{\text{ten}}$

c. $52_{\text{ten}} - 18_{\text{ten}}$

d. $75_{\text{ten}} - 36_{\text{ten}}$

e. $81_{\text{ten}} - 28_{\text{ten}}$

f. $100_{\text{ten}} - 3_{\text{ten}}$

g. $105_{\text{ten}} - 27_{\text{ten}}$

h. $411_{\text{ten}} - 272_{\text{ten}}$

i. $707_{\text{ten}} - 409_{\text{ten}}$

j. $1062_{\text{ten}} - 577_{\text{ten}}$

7. Find the answers to these subtraction problems.

a. $\begin{array}{r} 245_{\text{six}} \\ -154_{\text{six}} \\ \hline \end{array}$

b. $\begin{array}{r} 332_{\text{four}} \\ -23_{\text{four}} \\ \hline \end{array}$

c. $\begin{array}{r} 441_{\text{five}} \\ -44_{\text{five}} \\ \hline \end{array}$

d. $\begin{array}{r} 7744_{\text{nine}} \\ -365_{\text{nine}} \\ \hline \end{array}$

e. $\begin{array}{r} 2111_{\text{three}} \\ -22_{\text{three}} \\ \hline \end{array}$

8. Find the missing number in each case below.

a. $23_{\text{seven}} - ?_{\text{seven}} = 16_{\text{seven}}$

b. $?_{\text{four}} - 23_{\text{four}} = 2_{\text{four}}$

c. $15_{\text{nine}} - ?_{\text{nine}} = 5_{\text{nine}}$

d. $?_{\text{four}} - 11_{\text{four}} = 13_{\text{four}}$

e. $17_{\text{nine}} - ?_{\text{nine}} = 8_{\text{nine}}$

f. $?_{\text{six}} - 24_{\text{six}} = 15_{\text{six}}$

g. $?_{\text{ten}} - 14_{\text{ten}} = 47_{\text{ten}}$

h. $44_{\text{five}} - ?_{\text{five}} = 14_{\text{five}}$

i. $26_{\text{eight}} - ?_{\text{eight}} = 17_{\text{eight}}$

*9. Tell in what base each of these problems was done.

a. $14_? - 8_? = 6_?$

b. $12_? - 6_? = 3_?$

c. $15_? - 8_? = 6_?$

d. $35_? - 6_? = 27_?$

e. $11_? - 2_? = 2_?$

*10. Try each of these subtraction problems.

a. $40_{\text{twenty}} - 20_{\text{twenty}}$

b. $16_{\text{twelve}} - 12_{\text{twelve}}$

c. $30_{\text{sixteen}} - 9_{\text{sixteen}}$

d. $40_{\text{twelve}} - 15_{\text{twelve}}$

e. $52_{\text{fourteen}} - 49_{\text{fourteen}}$

*11. In base ten, when you add 6_{ten} to 4_{ten}, the sum is 10_{ten}. We call 6_{ten} a **complement** of 4_{ten}. Likewise, the complement of 8_{ten} is 2_{ten}. To subtract 8_{ten} from 12_{ten}, you can *add* the complement of 8_{ten} to 12_{ten}.

$$\begin{array}{r} 12_{\text{ten}} \\ +2_{\text{ten}} \\ \hline 14_{\text{ten}} \end{array}$$

Then, you subtract 10_{ten} from this sum. The answer to the original subtraction problem, $12_{\text{ten}} - 8_{\text{ten}}$, is 4_{ten}.

This method of subtracting is called the **complement method.** Use the complement method to find answers for each of the following.

a. $\begin{array}{r} 13_{\text{ten}} \\ -6_{\text{ten}} \\ \hline \end{array}$

b. $\begin{array}{r} 13_{\text{five}} \\ -4_{\text{five}} \\ \hline \end{array}$

c. $\begin{array}{r} 24_{\text{seven}} \\ -6_{\text{seven}} \\ \hline \end{array}$

d. $\begin{array}{r} 101_{\text{three}} \\ -2_{\text{three}} \\ \hline \end{array}$

e. $\begin{array}{r} 144_{\text{six}} \\ -25_{\text{six}} \\ \hline \end{array}$

THINK IT OVER

■ Each of these questions will cause you to think—and think *hard.* If you need to look back through the chapter to answer a particular question, do so.

1. At the beginning of this chapter, we discussed both addition and subtraction. How is subtraction related to addition?

2. You know that $4_{\text{five}} - 2_{\text{five}} = 2_{\text{five}}$ and $4_{\text{ten}} - 2_{\text{ten}} = 2_{\text{ten}}$. You also know that $11_{\text{five}} - 2_{\text{five}} = 4_{\text{five}}$ and $11_{\text{ten}} - 2_{\text{ten}} = 9_{\text{ten}}$. Why is it that for some subtraction problems the answer is the same as it would be in base ten, and for others the answer is different?

3. Look at problem 11 of the Try Your Hand section for this chapter. How does this method compare with the equal additions method of subtraction?

4. When you first learned how to subtract, you probably used the borrowing method. How is the borrowing method of subtraction related to the equal additions method? To the complement method?

4 A Look at Multiplication

LEARNING TO MULTIPLY

■ By now, you probably have a fairly comfortable feeling about working with numerals in other bases. You have learned to transform from one base to another and back again, and you have learned how to add and subtract in various bases.

In this chapter you are going to take a look at **multiplication** in other bases. Multiplication, you will find, is related to addition, and so this chapter on multiplication will be relatively easy to "conquer." You will find that what you learn in this chapter will be of great help when you finish this book and return to the base ten world you know so well.

Multiplication and Addition

1. In base ten, $3_{\text{ten}} \times 7_{\text{ten}}$ is the same as —?—.

a. $7_{\text{ten}} + 7_{\text{ten}} + 7_{\text{ten}}$ **b.** $7_{\text{ten}} \times 7_{\text{ten}} \times 7_{\text{ten}}$

a. $7_{\text{ten}} + 7_{\text{ten}} + 7_{\text{ten}}$

2. In base ten also, $2_{\text{ten}} \times 5_{\text{ten}}$ is the same as —?—.

a. $5_{\text{ten}} + 5_{\text{ten}}$ **b.** $5_{\text{ten}} \times 5_{\text{ten}}$

a. $5_{\text{ten}} + 5_{\text{ten}}$

3. That is, multiplication can be thought of as a form of ——?——.

a. division b. subtraction c. addition

c. addition

4. In later courses in mathematics, you'll learn different definitions of multiplication. In this book, however, we'll think of multiplication as a form of addition.

If you see $2_{five} \times 4_{five}$, you can think of it as $4_{five} + \_\_\_\__{five}$.

4

5. You know that $4_{five} + 4_{five} =$ ______ five(s) and ______ one(s).

1, 3

6. And in base five, 1 five and 3 ones is written as $\_\_\_\_\_\__{five}$.

13

7. Let's try an example in base six. You can write $2_{six} \times 5_{six}$ as $\_\_\_\__{six} + \_\_\_\__{six}$.

5, 5

8. In base six, $5_{six} + 5_{six} =$ ______ six(es) and ______ one(s).

1, 4

9. Then $2_{six} \times 5_{six} = \_\_\_\_\_\__{six}$.

14

10. You know that $2_{six} \times 3_{six} =$ ______ six(es) and ______ one(s).

1, 0

11. Then $2_{six} \times 3_{six} = \_\_\_\_\_\__{six}$.

10

12. In base four, $3_{four} \times 2_{four} =$ ______ four(s) and ______ one(s). 1, 2

13. In other words, $3_{four} \times 2_{four} =$ ______ $_{four}$. 12

14. Let's try 4×2 in several different bases:

$4_{five} \times 2_{five} =$ ______ five(s) and ______ one(s). 1, 3

Then $4_{five} \times 2_{five} =$ ______ $_{five}$. 13

15. $4_{six} \times 2_{six} =$ ______ six(es) and ______ one(s). 1, 2

Then $4_{six} \times 2_{six} =$ ______ $_{six}$. 12

16. $4_{seven} \times 2_{seven} =$ ______ seven(s) and ______ one(s). 1, 1

Then $4_{seven} \times 2_{seven} =$ ______ $_{seven}$. 11

17. $4_{eight} \times 2_{eight} =$ ______ eight(s) and ______ one(s). 1, 0

Then $4_{eight} \times 2_{eight} =$ ______ $_{eight}$. 10

18. $4_{nine} \times 2_{nine} =$ ______ nine(s) and ______ one(s). 0, 8

Then $4_{nine} \times 2_{nine} =$ ______ $_{nine}$. 8

19. Now let's try 3×3. In base ten, $3_{ten} \times 3_{ten} =$ ______ ten(s) 0

and ______ one(s). Then $3_{ten} \times 3_{ten} =$ ______ $_{ten}$. 9, 9

20. In base nine, $3_{nine} \times 3_{nine} =$ ______ nine(s) and ______ one(s). 1, 0

Then $3_{nine} \times 3_{nine} =$ ______ $_{nine}$. 10

21. And in base eight, $3_{\text{eight}} \times 3_{\text{eight}} =$ ______ eight(s) and ______ one(s). Then $3_{\text{eight}} \times 3_{\text{eight}} =$ ______ $_{\text{eight}}$. 1, 1 11

22. In the next few frames, we're going to multiply in base five. Let's start with 1_{five}:

1. $1_{\text{five}} \times 0_{\text{five}} =$ ____ $_{\text{five}}$ 0
2. $1_{\text{five}} \times 1_{\text{five}} =$ ____ $_{\text{five}}$ 1
3. $1_{\text{five}} \times 2_{\text{five}} =$ ____ $_{\text{five}}$ 2
4. $1_{\text{five}} \times 3_{\text{five}} =$ ____ $_{\text{five}}$ 3
5. $1_{\text{five}} \times 4_{\text{five}} =$ ____ $_{\text{five}}$ 4

23. Still in base five, let's multiply by 2_{five}:

1. $2_{\text{five}} \times 0_{\text{five}} =$ ______ $_{\text{five}}$ 0
2. $2_{\text{five}} \times 1_{\text{five}} =$ ______ $_{\text{five}}$ 2
3. $2_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$ 4
4. $2_{\text{five}} \times 3_{\text{five}} =$ ______ $_{\text{five}}$ 11
5. $2_{\text{five}} \times 4_{\text{five}} =$ ______ $_{\text{five}}$ 13

24. $2_{\text{five}} \times 4_{\text{five}}$ is written 13_{five} because $2_{\text{five}} \times 4_{\text{five}}$ is ______ five(s) and ______ one(s). 1 3

25. Let's multiply by 3_{five} in base five:

1. $3_{\text{five}} \times 0_{\text{five}} =$ ______ $_{\text{five}}$ 0
2. $3_{\text{five}} \times 1_{\text{five}} =$ ______ $_{\text{five}}$ 3
3. $3_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$ 11
4. $3_{\text{five}} \times 3_{\text{five}} =$ ______ $_{\text{five}}$ 14
5. $3_{\text{five}} \times 4_{\text{five}} =$ ______ $_{\text{five}}$ 22

26. $3_{\text{five}} \times 4_{\text{five}} = 22_{\text{five}}$ because $3_{\text{five}} \times 4_{\text{five}}$ is ______ five(s) and ______ one(s). 2 2

27. Let's continue multiplying in base five:

1. $4_{five} \times 0_{five} =$ ______ $_{five}$ — 0

2. $4_{five} \times 1_{five} =$ ______ $_{five}$ — 4

3. $4_{five} \times 2_{five} =$ ______ $_{five}$ — 13

4. $4_{five} \times 3_{five} =$ ______ $_{five}$ — 22

5. $4_{five} \times 4_{five} =$ ______ $_{five}$ — 31

28. Now let's try another base. In base seven, $5_{seven} \times 6_{seven} =$ ______ seven(s) and ______ one(s). That is, $5_{seven} \times 6_{seven} =$ ______ $_{seven}$. — 4; 2, 42

29. In base six, $3_{six} \times 4_{six} =$ ______ $_{six}$. — 20

30. In base four, $2_{four} \times 3_{four} =$ ______ $_{four}$. — 12

31. Try these multiplication problems in various bases:

1. $5_{eight} \times 4_{eight} =$ ______ $_{eight}$ — 24

2. $3_{seven} \times 5_{seven} =$ ______ $_{seven}$ — 21

3. $2_{three} \times 2_{three} =$ ______ $_{three}$ — 11

4. $4_{five} \times 4_{five} =$ ______ $_{five}$ — 31

5. $8_{ten} \times 7_{ten} =$ ______ $_{ten}$ — 56

Extending Your Multiplication Knowledge

32. So far, you have found the product of two numbers when both numbers were less than the base.

In this section we're going to use the base itself as a multiplier. You remember that 10_{five} means —?—.

a. one **b.** ten **c.** five — c. five

33. And 10_{six} means ——?——.

a. one   b. ten   c. six   c. six

34. Since multiplication can be thought of as a form of addition, $2_{five} \times 10_{five}$ means ——?——.

a. $10_{five} + 10_{five}$ b. $2_{five} + 2_{five}$ c. $10_{five} + 2_{five}$ a. $10_{five} + 10_{five}$

35. You know that $10_{five} + 10_{five} =$ ______ $_{five}$. Thus, you can see that   20

$2_{five} \times 10_{five} =$ ______ $_{five}$.   20

36. And since $10_{six} + 10_{six} = 20_{six}$, it is easy to see that

$2_{six} \times 10_{six} =$ ______ $_{six}$.   20

37. Earlier you saw that

$3_{five} \times 1_{five} =$ ______ $_{five}$.   3

Then it follows that

$3_{five} \times 10_{five} =$ ______ $_{five}$.   30

38. In base six:

$4_{six} \times 1_{six} = 4_{six}$.

$4_{six} \times 10_{six} =$ ______ $_{six}$.   40

39. In base seven, $5_{seven} \times 10_{seven} =$ ______ $_{seven}$.   50

40. Try each of the following multiplication problems:

1. $3_{eight} \times 10_{eight} =$ ______ eight   30
2. $6_{seven} \times 10_{seven} =$ ______ seven   60
3. $2_{three} \times 10_{three} =$ ______ three   20
4. $5_{ten} \times 10_{ten} =$ ______ ten   50
5. $4_{six} \times 10_{six} =$ ______ six   40

12. In base four, $3_{\text{four}} \times 2_{\text{four}} =$ ______ four(s) and ______ one(s). 1, 2

13. In other words, $3_{\text{four}} \times 2_{\text{four}} =$ ______ $_{\text{four}}$. 12

14. Let's try 4×2 in several different bases:

$4_{\text{five}} \times 2_{\text{five}} =$ ______ five(s) and ______ one(s). 1, 3

Then $4_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$. 13

15. $4_{\text{six}} \times 2_{\text{six}} =$ ______ six(es) and ______ one(s). 1, 2

Then $4_{\text{six}} \times 2_{\text{six}} =$ ______ $_{\text{six}}$. 12

16. $4_{\text{seven}} \times 2_{\text{seven}} =$ ______ seven(s) and ______ one(s). 1, 1

Then $4_{\text{seven}} \times 2_{\text{seven}} =$ ______ $_{\text{seven}}$. 11

17. $4_{\text{eight}} \times 2_{\text{eight}} =$ ______ eight(s) and ______ one(s). 1, 0

Then $4_{\text{eight}} \times 2_{\text{eight}} =$ ______ $_{\text{eight}}$. 10

18. $4_{\text{nine}} \times 2_{\text{nine}} =$ ______ nine(s) and ______ one(s). 0, 8

Then $4_{\text{nine}} \times 2_{\text{nine}} =$ ______ $_{\text{nine}}$. 8

19. Now let's try 3×3. In base ten, $3_{\text{ten}} \times 3_{\text{ten}} =$ ______ ten(s) 0

and ______ one(s). Then $3_{\text{ten}} \times 3_{\text{ten}} =$ ____ $_{\text{ten}}$. 9, 9

20. In base nine, $3_{\text{nine}} \times 3_{\text{nine}} =$ ______ nine(s) and ______ one(s). 1, 0

Then $3_{\text{nine}} \times 3_{\text{nine}} =$ ______ $_{\text{nine}}$. 10

21. And in base eight, $3_{\text{eight}} \times 3_{\text{eight}} =$ ______ eight(s) and ______ one(s). Then $3_{\text{eight}} \times 3_{\text{eight}} =$ ______ $_{\text{eight}}$.

1, 1
11

22. In the next few frames, we're going to multiply in base five. Let's start with 1_{five}:

1. $1_{\text{five}} \times 0_{\text{five}} =$ ____ $_{\text{five}}$ 0
2. $1_{\text{five}} \times 1_{\text{five}} =$ ____ $_{\text{five}}$ 1
3. $1_{\text{five}} \times 2_{\text{five}} =$ ____ $_{\text{five}}$ 2
4. $1_{\text{five}} \times 3_{\text{five}} =$ ____ $_{\text{five}}$ 3
5. $1_{\text{five}} \times 4_{\text{five}} =$ ____ $_{\text{five}}$ 4

23. Still in base five, let's multiply by 2_{five}:

1. $2_{\text{five}} \times 0_{\text{five}} =$ ______ $_{\text{five}}$ 0
2. $2_{\text{five}} \times 1_{\text{five}} =$ ______ $_{\text{five}}$ 2
3. $2_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$ 4
4. $2_{\text{five}} \times 3_{\text{five}} =$ ______ $_{\text{five}}$ 11
5. $2_{\text{five}} \times 4_{\text{five}} =$ ______ $_{\text{five}}$ 13

24. $2_{\text{five}} \times 4_{\text{five}}$ is written 13_{five} because $2_{\text{five}} \times 4_{\text{five}}$ is ______ five(s) and ______ one(s).

1
3

25. Let's multiply by 3_{five} in base five:

1. $3_{\text{five}} \times 0_{\text{five}} =$ ______ $_{\text{five}}$ 0
2. $3_{\text{five}} \times 1_{\text{five}} =$ ______ $_{\text{five}}$ 3
3. $3_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$ 11
4. $3_{\text{five}} \times 3_{\text{five}} =$ ______ $_{\text{five}}$ 14
5. $3_{\text{five}} \times 4_{\text{five}} =$ ______ $_{\text{five}}$ 22

26. $3_{\text{five}} \times 4_{\text{five}} = 22_{\text{five}}$ because $3_{\text{five}} \times 4_{\text{five}}$ is ______ five(s) and ______ one(s).

2
2

27. Let's continue multiplying in base five:

1. $4_{five} \times 0_{five} = \_\_\_\_\__{five}$ — 0
2. $4_{five} \times 1_{five} = \_\_\_\_\__{five}$ — 4
3. $4_{five} \times 2_{five} = \_\_\_\_\__{five}$ — 13
4. $4_{five} \times 3_{five} = \_\_\_\_\__{five}$ — 22
5. $4_{five} \times 4_{five} = \_\_\_\_\__{five}$ — 31

28. Now let's try another base. In base seven, $5_{seven} \times 6_{seven} = \_\_\_\_\_$ seven(s) and _____ one(s). That is, $5_{seven} \times 6_{seven} = \_\_\_\_\__{seven}$. — 4; 2, 42

29. In base six, $3_{six} \times 4_{six} = \_\_\_\_\__{six}$. — 20

30. In base four, $2_{four} \times 3_{four} = \_\_\_\_\__{four}$. — 12

31. Try these multiplication problems in various bases:

1. $5_{eight} \times 4_{eight} = \_\_\_\_\__{eight}$ — 24
2. $3_{seven} \times 5_{seven} = \_\_\_\_\__{seven}$ — 21
3. $2_{three} \times 2_{three} = \_\_\_\_\__{three}$ — 11
4. $4_{five} \times 4_{five} = \_\_\_\_\__{five}$ — 31
5. $8_{ten} \times 7_{ten} = \_\_\_\_\__{ten}$ — 56

Extending Your Multiplication Knowledge

32. So far, you have found the product of two numbers when both numbers were less than the base.

In this section we're going to use the base itself as a multiplier. You remember that 10_{five} means —?—.

a. one **b.** ten **c.** five — c. five

33. And 10_{six} means ——?——.

a. one b. ten c. six

c. six

34. Since multiplication can be thought of as a form of addition, $2_{five} \times 10_{five}$ means ——?——.

a. $10_{five} + 10_{five}$ b. $2_{five} + 2_{five}$ c. $10_{five} + 2_{five}$

a. $10_{five} + 10_{five}$

35. You know that $10_{five} + 10_{five} =$ ______ $_{five}$. Thus, you can see that 20

$2_{five} \times 10_{five} =$ ______ $_{five}$. 20

36. And since $10_{six} + 10_{six} = 20_{six}$, it is easy to see that

$2_{six} \times 10_{six} =$ ______ $_{six}$. 20

37. Earlier you saw that

$3_{five} \times 1_{five} =$ ______ $_{five}$. 3

Then it follows that

$3_{five} \times 10_{five} =$ ______ $_{five}$. 30

38. In base six:

$4_{six} \times 1_{six} = 4_{six}$.

$4_{six} \times 10_{six} =$ ______ $_{six}$. 40

39. In base seven, $5_{seven} \times 10_{seven} =$ ______ $_{seven}$. 50

40. Try each of the following multiplication problems:

1. $3_{eight} \times 10_{eight} =$ ______ $_{eight}$ 30
2. $6_{seven} \times 10_{seven} =$ ______ $_{seven}$ 60
3. $2_{three} \times 10_{three} =$ ______ $_{three}$ 20
4. $5_{ten} \times 10_{ten} =$ ______ $_{ten}$ 50
5. $4_{six} \times 10_{six} =$ ______ $_{six}$ 40

41. In base five, you remember that

$$2_{\text{five}} \times 2_{\text{five}} = 4_{\text{five}}.$$

Then it is easy to see that

$$2_{\text{five}} \times 20_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}.$$ 40

42. Since in base four,

$$3_{\text{four}} \times 2_{\text{four}} = 12_{\text{four}},$$

you can see that

$$3_{\text{four}} \times 20_{\text{four}} = \_\_\_\_\_\_ _{\text{four}}.$$ 120

43. In base eight:

$$3_{\text{eight}} \times 2_{\text{eight}} = 6_{\text{eight}}$$

$$3_{\text{eight}} \times 20_{\text{eight}} = \_\_\_\_\_\_ _{\text{eight}}$$ 60

44. In base five:

$$4_{\text{five}} \times 2_{\text{five}} = 13_{\text{five}}$$

$$4_{\text{five}} \times 20_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$$ 130

45. In base nine:

$$8_{\text{nine}} \times 2_{\text{nine}} = 17_{\text{nine}}$$

$$8_{\text{nine}} \times 20_{\text{nine}} = \_\_\_\_\_\_ _{\text{nine}}$$ 170

46. Try these multiplication problems in base five:

1. $2_{\text{five}} \times 20_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$ 40
2. $3_{\text{five}} \times 20_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$ 110
3. $3_{\text{five}} \times 30_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$ 140
4. $4_{\text{five}} \times 30_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$ 220
5. $2_{\text{five}} \times 30_{\text{five}} = \_\_\_\_\_\_ _{\text{five}}$ 110

47. Now try these multiplication problems:

1. $4_{\text{nine}} \times 20_{\text{nine}} = \_\_\_\_\_\_ _{\text{nine}}$ 80
2. $3_{\text{seven}} \times 30_{\text{seven}} = \_\_\_\_\_\_ _{\text{seven}}$ 120
3. $2_{\text{three}} \times 20_{\text{three}} = \_\_\_\_\_\_ _{\text{three}}$ 110
4. $9_{\text{ten}} \times 90_{\text{ten}} = \_\_\_\_\_\_ _{\text{ten}}$ 810
5. $6_{\text{seven}} \times 40_{\text{seven}} = \_\_\_\_\_\_ _{\text{seven}}$ 330

48. The principle we are using can be extended to include problems like $2_{\text{five}} \times 100_{\text{five}}$. You know that

$$2_{\text{five}} \times 1_{\text{five}} = 2_{\text{five}}$$

and

$$2_{\text{five}} \times 10_{\text{five}} = 20_{\text{five}}.$$

Then

$$2_{\text{five}} \times 100_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}.$$ 200

49. $3_{\text{six}} \times 100_{\text{six}} =$ —?—.

a. 3_{six} b. 30_{six} c. 300_{six} c. 300_{six}

50. $4_{\text{five}} \times 1000_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$. 4000

51. Try each of the following:

1. $3_{\text{six}} \times 1000_{\text{six}} = \underline{\qquad\qquad}_{\text{six}}$ 3000
2. $2_{\text{eight}} \times 2000_{\text{eight}} = \underline{\qquad\qquad}_{\text{eight}}$ 4000
3. $5_{\text{nine}} \times 100_{\text{nine}} = \underline{\qquad\qquad}_{\text{nine}}$ 500
4. $4_{\text{ten}} \times 200_{\text{ten}} = \underline{\qquad\qquad}_{\text{ten}}$ 800
5. $3_{\text{seven}} \times 200_{\text{seven}} = \underline{\qquad\qquad}_{\text{seven}}$ 600

52. In base five,

$$4_{\text{five}} \times 2_{\text{five}} = 13_{\text{five}}.$$

Then it follows that

$$4_{\text{five}} \times 200_{\text{five}} = \text{—?—}.$$

a. 800_{five} b. 1300_{five} c. 13000_{five} b. 1300_{five}

53. In base five, $3_{\text{five}} \times 3000_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$. 14000

54. In base six,

$$4_{\text{six}} \times 5_{\text{six}} = 32_{\text{six}}$$

Then you can see that

$$4_{\text{six}} \times 5000_{\text{six}} = \underline{\hspace{3cm}}_{\text{six}}.$$ 32000

55. In base eight:

$$7_{\text{eight}} \times 6_{\text{eight}} = 52_{\text{eight}}$$

$$7_{\text{eight}} \times 600_{\text{eight}} = \underline{\hspace{3cm}}_{\text{eight}}$$ 5200

56. In base nine:

$$3_{\text{nine}} \times 3_{\text{nine}} = 10_{\text{nine}}$$

$$3_{\text{nine}} \times 300_{\text{nine}} = \underline{\hspace{3cm}}_{\text{nine}}$$ 1000

57. Try each of the following multiplication problems:

1. $3_{\text{nine}} \times 800_{\text{nine}} = \underline{\hspace{3cm}}_{\text{nine}}$ 2600
2. $4_{\text{seven}} \times 60_{\text{seven}} = \underline{\hspace{3cm}}_{\text{seven}}$ 330
3. $2_{\text{eight}} \times 400_{\text{eight}} = \underline{\hspace{3cm}}_{\text{eight}}$ 1000
4. $5_{\text{ten}} \times 2000_{\text{ten}} = \underline{\hspace{3cm}}_{\text{ten}}$ 10000
5. $4_{\text{five}} \times 400_{\text{five}} = \underline{\hspace{3cm}}_{\text{five}}$ 3100

58. Now let's extend the multiplication principle once again. You learned earlier that

$$1_{\text{five}} \times 1_{\text{five}} = 1_{\text{five}},$$

and

$$1_{\text{five}} \times 10_{\text{five}} = 10_{\text{five}}.$$

Then it follows that

$$10_{\text{five}} \times 10_{\text{five}} = \text{——?——}.$$

a. 10_{five} b. 100_{five} c. 400_{five} b. 100_{five}

59. In base eight:

$$1_{\text{eight}} \times 1_{\text{eight}} = 1_{\text{eight}}$$

$$1_{\text{eight}} \times 10_{\text{eight}} = 10_{\text{eight}}$$

$$10_{\text{eight}} \times 10_{\text{eight}} = \underline{\hspace{3cm}}_{\text{eight}}$$ 100

60. In base six, $10_{six} \times 10_{six} =$ ________ $_{six}$. 100

61. And also $100_{six} \times 10_{six} =$ ________ $_{six}$. 1000

62. In base five,

$$2_{five} \times 2_{five} = 4_{five},$$

and

$$2_{five} \times 20_{five} = 40_{five}.$$

Then it is easy to see that

$$20_{five} \times 20_{five} = \_\_\_\_\_\_\_\_ _{five}.$$ 400

63. Also in base five,

$$3_{five} \times 4_{five} = 22_{five},$$

and

$$3_{five} \times 40_{five} = 220_{five}.$$

Then it follows that

$$30_{five} \times 40_{five} = \_\_\_\_\_\_\_\_ _{five}.$$ 2200

64. In base seven:

$$5_{seven} \times 3_{seven} = 21_{seven}$$

$$50_{seven} \times 30_{seven} = \_\_\_\_\_\_\_\_ _{seven}$$ 2100

65. In base ten:

$$4_{ten} \times 3_{ten} = 12_{ten}$$

$$40_{ten} \times 30_{ten} = \_\_\_\_\_\_\_\_ _{ten}$$ 1200

66. In base eight:

$$3_{eight} \times 2_{eight} = 6_{eight}$$

$$300_{eight} \times 20_{eight} = \_\_\_\_\_\_\_\_ _{eight}$$ 6000

67. In base seven, $200_{seven} \times 2_{seven} =$ ________ $_{seven}$. 400

$50 \times 30 = ??00$

68. In base six, $50_{six} \times 30_{six} =$ ________ $_{six}$. 2300

69. In base three, $200_{three} \times 20_{three} =$ ________ $_{three}$. 11000

70. In base four, $20_{four} \times 20_{four} =$ ________ $_{four}$. 1000

71. Complete the blanks in the following base five multiplication examples:

1. $10_{five} \times 1000_{five} =$ ________ $_{five}$ 10000
2. $40_{five} \times 20_{five} =$ ________ $_{five}$ 1300
3. $100_{five} \times 30_{five} =$ ________ $_{five}$ 3000
4. $30_{five} \times 300_{five} =$ ________ $_{five}$ 14000
5. $40_{five} \times 3000_{five} =$ ________ $_{five}$ 220000

72. Now try the following problems:

1. $40_{nine} \times 20_{nine} =$ ________ $_{nine}$ 800
2. $70_{eight} \times 20_{eight} =$ ________ $_{eight}$ 1600
3. $30_{four} \times 100_{four} =$ ________ $_{four}$ 3000
4. $700_{ten} \times 400_{ten} =$ ________ $_{ten}$ 280000
5. $200_{three} \times 20_{three} =$ ________ $_{three}$ 11000

Learning About Partial Products

73. In this section we are going to take a look at multiplication problems where one or both of the numbers exceed the base.

One of the problems of this type is $2_{five} \times 13_{five}$. If we think of multiplication as a form of addition, we can say that

$2_{five} \times 13_{five} = \_\_\_\_\__{five} + \_\_\_\_\__{five}$. 13, 13

74. And since $13_{five} + 13_{five} = \_\_\_\_\__{five}$, $2_{five} \times 13_{five}$ is also equal 31

to $\_\_\_\_\__{five}$. 31

75. Now here's a different way of looking at the same problem, $2_{five} \times 13_{five}$.

First we can say that 13_{five} is $10_{five} + \_\_\_\_\__{five}$. 3

76. Then we have:

$2_{five} \times 10_{five} = \_\_\_\_\__{five}$ 20

$2_{five} \times 3_{five} = \_\_\_\_\__{five}$ 11

77. Adding the two **partial products**, we see that

$2_{five} \times 13_{five} = \_\_\_\_\__{five}$. 31

78. To multiply 23_{five} by 2_{five} we can write 23_{five} as $20_{five} + \_\_\_\_\__{five}$. 3

79. For convenience we will write the smaller partial product first. Thus, we have:

$2_{five} \times 3_{five} = \_\_\_\_\__{five}$ 11

$2_{five} \times 20_{five} = \_\_\_\_\__{five}$ 40

80. The sum of the partial products is $\_\_\_\__{\text{five}}$, and thus 101

$$2_{\text{five}} \times 23_{\text{five}} = \_\_\_\__{\text{five}}.$$ 101

81. Let's multiply 14_{five} by 2_{five}.

1. First we write 14_{five} as $10_{\text{five}} + \_\_\_\__{\text{five}}$. 4
2. Then we multiply both parts by 2_{five} to find partial products:

$$2_{\text{five}} \times 4_{\text{five}} = \_\_\_\__{\text{five}}$$ 13

$$2_{\text{five}} \times 10_{\text{five}} = \_\_\_\__{\text{five}}$$ 20

3. And we add the partial products:

$$13_{\text{five}} + 20_{\text{five}} = \_\_\_\__{\text{five}}$$ 33

4. Thus we see that

$$2_{\text{five}} \times 14_{\text{five}} = \_\_\_\__{\text{five}}.$$ 33

82. Suppose you were asked to multiply 24_{five} by 4_{five}.

1. First you would rewrite 24_{five} as $\_\_\_\__{\text{five}} + \_\_\_\__{\text{five}}$. 20, 4 (or 4, 20. You can write the numerals in either order.)
2. Then you would have two partial products:

$$4_{\text{five}} \times 4_{\text{five}} = \_\_\_\__{\text{five}}$$ 31

$$4_{\text{five}} \times 20_{\text{five}} = \_\_\_\__{\text{five}}$$ 130

3. And you would add the partial products:

$$31_{\text{five}} + 130_{\text{five}} = \_\_\_\__{\text{five}}$$ 211

4. You would find out that:

$$4_{\text{five}} \times 24_{\text{five}} = \_\_\_\__{\text{five}}.$$ 211

83. Let's try one in base eight. To multiply 25_{eight} by 3_{eight}, we write 25_{eight} as $\_\_\_\__{\text{eight}} + 5_{\text{eight}}$. Then we multiply both 5_{eight} and 20_{eight} by $\_\_\_\__{\text{eight}}$. The two partial products are $\_\_\_\__{\text{eight}}$ and $\_\_\_\__{\text{eight}}$. Adding the partial products, we find that 20 | 3, 17 | 60

$$3_{\text{eight}} \times 25_{\text{eight}} = \_\_\_\__{\text{eight}}.$$ 77

84. In base four, you would multiply 31_{four} by 2_{four} by finding the partial products:

$$2_{\text{four}} \times 1_{\text{four}} = \_\_\_\__{\text{four}}$$ 2

$$2_{\text{four}} \times 30_{\text{four}} = \_\_\_\__{\text{four}}.$$ 120

Then $$2_{\text{four}} \times 31_{\text{four}} = \_\_\_\__{\text{four}}.$$ 122

85. In base ten, to multiply 47_{ten} by 5_{ten}, you would find these partial products:

$5_{\text{ten}} \times 7_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}$ 35

$5_{\text{ten}} \times 40_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}.$ 200

Then $5_{\text{ten}} \times 47_{\text{ten}} = \_\_\_\_\_\__{\text{ten}}.$ 235

86. Just for the sake of practice, here are some exercises in writing partial products:

1. $5_{\text{seven}} \times 46_{\text{seven}} = \begin{cases} \_\_\_\__{\text{seven}} \times 6_{\text{seven}} \\ \_\_\_\__{\text{seven}} \times 40_{\text{seven}} \end{cases}$ 5 5

2. $4_{\text{five}} \times 31_{\text{five}} = \begin{cases} 4_{\text{five}} \times \_\_\_\__{\text{five}} \\ \_\_\_\__{\text{five}} \times 30_{\text{five}} \end{cases}$ 1 4

3. $75_{\text{ten}} \times 6_{\text{ten}} = \begin{cases} 5_{\text{ten}} \times \_\_\_\__{\text{ten}} \\ \_\_\_\__{\text{ten}} \times 6_{\text{ten}} \end{cases}$ 6 70

4. $21_{\text{three}} \times 2_{\text{three}} = \begin{cases} 1_{\text{three}} \times \_\_\_\__{\text{three}} \\ \_\_\_\__{\text{three}} \times 2_{\text{three}} \end{cases}$ 2 20

(Did you see that we switched the numerals around? We are still listing the smaller partial product first.)

5. $87_{\text{nine}} \times 6_{\text{nine}} = \begin{cases} \_\_\_\__{\text{nine}} \times 6_{\text{nine}} \\ 80_{\text{nine}} \times \_\_\_\__{\text{nine}} \end{cases}$ 7 6

87. In base six:

$$43_{\text{six}} \times 5_{\text{six}} = \begin{cases} 3_{\text{six}} \times 5_{\text{six}} = \_\_\_\_\_\__{\text{six}} \\ 40_{\text{six}} \times 5_{\text{six}} = \_\_\_\_\_\__{\text{six}} \end{cases}$$ 23 320

The sum of the partial products is $\_\_\_\_\_\__{\text{six}}.$ 343

88. In base eight:

$$6_{\text{eight}} \times 27_{\text{eight}} = \begin{cases} 6_{\text{eight}} \times 7_{\text{eight}} = \_\_\_\_\_\__{\text{eight}} \\ 6_{\text{eight}} \times 20_{\text{eight}} = \_\_\_\_\_\__{\text{eight}} \end{cases}$$ 52 140

The sum of the partial products is $\_\_\_\_\_\__{\text{eight}}$, and thus 212

$6_{\text{eight}} \times 27_{\text{eight}} = \_\_\_\_\_\__{\text{eight}}.$ 212

89. In these next few frames, perform the necessary computations on scrap paper.

When 27_{six} is multiplied by 5_{six}, the partial products are

______ $_{\text{six}}$ and ______ $_{\text{six}}$. Then $27_{\text{six}} \times 5_{\text{six}} =$ ______ $_{\text{six}}$. 55, 140, 235

90. When 7_{ten} is multiplied by 64_{ten}, the partial products are

______ $_{\text{ten}}$ and ______ $_{\text{ten}}$. Then $7_{\text{ten}} \times 64_{\text{ten}} =$ ______ $_{\text{ten}}$. 28, 420, 448

91. When 37_{eight} is multiplied by 5_{eight}, the partial products are

______ $_{\text{eight}}$ and ______ $_{\text{eight}}$. Then $37_{\text{eight}} \times 5_{\text{eight}} =$ ______ $_{\text{eight}}$. 43, 170, 233

92. $3_{\text{five}} \times 41_{\text{five}} =$ ______ $_{\text{five}}$. (If necessary, use scrap paper for the computations.) 223

93. $22_{\text{four}} \times 3_{\text{four}} =$ ______ $_{\text{four}}$. 132

94. $2_{\text{three}} \times 12_{\text{three}} =$ ______ $_{\text{three}}$. 101

95. Our method of multiplying may be used even when there are more than two partial products.

To multiply 123_{five} by 2_{five}, we first write 123_{five} as

$100_{\text{five}} + 20_{\text{five}} +$ ______ $_{\text{five}}$. 3

96. Then we multiply by 2_{five} to get our partial products:

$2_{\text{five}} \times 3_{\text{five}} =$ ______ $_{\text{five}}$ 11

$2_{\text{five}} \times 20_{\text{five}} =$ ______ $_{\text{five}}$ 40

$2_{\text{five}} \times 100_{\text{five}} =$ ______ $_{\text{five}}$ 200

97. The sum of the partial products is ______ $_{\text{five}}$, and thus 301

$123_{\text{five}} \times 2_{\text{five}} =$ ______ $_{\text{five}}$. 301

98. Let's try $3_{\text{five}} \times 114_{\text{five}}$:

$3_{\text{five}} \times \ 4_{\text{five}} =$ ________ five — 22

$3_{\text{five}} \times \ 10_{\text{five}} =$ ________ five — 30

$3_{\text{five}} \times 100_{\text{five}} =$ ________ five — 300

The sum of the partial products is ________ five. — 402

99. In base ten, $4_{\text{ten}} \times 132_{\text{ten}}$ gives the following partial products:

$4_{\text{ten}} \times \ 2_{\text{ten}} =$ ________ ten — 8

$4_{\text{ten}} \times \ 30_{\text{ten}} =$ ________ ten — 120

$4_{\text{ten}} \times 100_{\text{ten}} =$ ________ ten — 400

The sum of the partial products is ________ ten. — 528

100. In multiplying 346_{seven} by 4_{seven}, there are three partial products:

$4_{\text{seven}} \times \ 6_{\text{seven}} =$ ________ seven — 33

$4_{\text{seven}} \times \ 40_{\text{seven}} =$ ________ seven — 220

$4_{\text{seven}} \times 300_{\text{seven}} =$ ________ seven — 1500

Then $346_{\text{seven}} \times 4_{\text{seven}} =$ ________ seven. — 2053

101. Let's try $2_{\text{five}} \times 403_{\text{five}}$:

$2_{\text{five}} \times \ 3_{\text{five}} =$ ________ five — 11

$2_{\text{five}} \times \ 0_{\text{five}} =$ ________ five — 0

$2_{\text{five}} \times 400_{\text{five}} =$ ________ five — 1300

Then $2_{\text{five}} \times 403_{\text{five}} =$ ________ five. — 1311

102. In base ten, there are three partial products for $6_{\text{ten}} \times 527_{\text{ten}}$:

$6_{\text{ten}} \times 7_{\text{ten}} =$ ________ ten — 42

$6_{\text{ten}} \times 20_{\text{ten}} =$ ________ ten — 120

$6_{\text{ten}} \times 500_{\text{ten}} =$ ________ ten — 3000

Then $6_{\text{ten}} \times 527_{\text{ten}} =$ ________ ten. — 3162

103. Complete the blanks and find the answer to $3_{\text{five}} \times 421_{\text{five}}$:

$3_{\text{five}} \times 1_{\text{five}} =$ ________ five — 3

$3_{\text{five}} \times$ ________ five $= 110_{\text{five}}$ — 20

$3_{\text{five}} \times$ ________ five $=$ ________ five — 400, 2200

The sum of the partial products is ________ five. — 2313

104. In base five, try $3_{\text{five}} \times 114_{\text{five}}$:

____ five $\times$ ________ five $=$ ________ five — $3_{\text{five}} \times 4_{\text{five}} = 22_{\text{five}}$

____ five $\times$ ________ five $=$ ________ five — $3_{\text{five}} \times 10_{\text{five}} = 30_{\text{five}}$

____ five $\times$ ________ five $=$ ________ five — $3_{\text{five}} \times 100_{\text{five}} = 300_{\text{five}}$

Then $3_{\text{five}} \times 114_{\text{five}} =$ ________ five. — 402

105. In base ten, try $6_{\text{ten}} \times 317_{\text{ten}}$:

____ ten $\times$ ________ ten $=$ ________ ten — $6_{\text{ten}} \times 7_{\text{ten}} = 42_{\text{ten}}$

____ ten $\times$ ________ ten $=$ ________ ten — $6_{\text{ten}} \times 10_{\text{ten}} = 60_{\text{ten}}$

____ ten $\times$ ________ ten $=$ ________ ten — $6_{\text{ten}} \times 300_{\text{ten}} = 1800_{\text{ten}}$

Then $6_{\text{ten}} \times 317_{\text{ten}} =$ ________ ten. — 1902

106. $4_{\text{five}} \times 433_{\text{five}} =$ ________ five. (Do the computation on scrap paper.) — 3342

107. $2_{\text{five}} \times 1421_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$. 3342

108. $4_{\text{five}} \times 1032_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$. 4233

Using Partial Products

109. When we multiply 2_{five} by 13_{five}, we write 13_{five} as $3_{\text{five}} + 10_{\text{five}}$. Then we multiply 2_{five} by both 3_{five} and 10_{five}. Finally, we add the partial products to get an answer.

Now, we're going to use the partial products method to multiply 12_{five} by 23_{five}. First, we'll write 23_{five} as $\underline{\qquad}_{\text{five}} + 20_{\text{five}}$. 3

Then we have two partial products:

$12_{\text{five}} \times 3_{\text{five}}$

$12_{\text{five}} \times \underline{\qquad\qquad}_{\text{five}}$ 20

110. And each of these can be written as partial products:

$$12_{\text{five}} \times 3_{\text{five}} = \begin{cases} 2_{\text{five}} \times 3_{\text{five}} \\ 10_{\text{five}} \times 3_{\text{five}} \end{cases}$$

$$12_{\text{five}} \times 20_{\text{five}} = \begin{cases} 2_{\text{five}} \times 20_{\text{five}} \\ \underline{\qquad\qquad}_{\text{five}} \times 20_{\text{five}} \end{cases}$$ 10

111. Then we have four partial products:

$2_{\text{five}} \times 3_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$ 11

$10_{\text{five}} \times 3_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$ 30

$2_{\text{five}} \times 20_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$ 40

$10_{\text{five}} \times 20_{\text{five}} = \underline{\qquad\qquad}_{\text{five}}$ 200

The sum of the partial products is $\underline{\qquad\qquad}_{\text{five}}$, and thus the 331

answer to the original problem, $12_{\text{five}} \times 23_{\text{five}}$, is $\underline{\qquad\qquad}_{\text{five}}$. 331

112. Let's try $12_{five} \times 34_{five}$. We can write 34_{five} as $30_{five} + \_\_\_\__{five}$. Then there are two partial products: 4

$$12_{five} \times 4_{five}$$
$$12_{five} \times 30_{five}$$

Each of these can be written as two partial products since 12_{five} can be expressed as $10_{five} + \_\_\_\__{five}$. The four partial products are: 2

$$12_{five} \times 4_{five} = \begin{cases} 2_{five} \times 4_{five} \\ 10_{five} \times 4_{five} \end{cases}$$

$$12_{five} \times 30_{five} = \begin{cases} 2_{five} \times 30_{five} \\ 10_{five} \times \_\_\_\__{five} \end{cases}$$ 30

113. Then, to find the product of 12_{five} and 34_{five}, we write:

$2_{five} \times 4_{five} = \_\_\_\__{five}$ 13

$10_{five} \times 4_{five} = \_\_\_\__{five}$ 40

$2_{five} \times 30_{five} = \_\_\_\__{five}$ 110

$10_{five} \times 30_{five} = \_\_\_\__{five}$ 300

The sum of the partial products gives the product of 12_{five} and 34_{five}. The product of 12_{five} and 34_{five} is $\_\_\_\__{five}$. 1013

114. For the multiplication problem $24_{nine} \times 18_{nine}$, the partial products are:

$4_{nine} \times 8_{nine}$

$20_{nine} \times 8_{nine}$

$4_{nine} \times \_\_\_\__{nine}$ 10

$20_{nine} \times \_\_\_\__{nine}$ 10

115. To find the answer to $24_{five} \times 31_{five}$, you would write the following partial products:

$4_{five} \times \_\_\_\__{five}$ 1

$4_{five} \times \_\_\_\__{five}$ 30

$20_{five} \times \_\_\_\__{five}$ 1

$20_{five} \times \_\_\_\__{five}$ 30

116. The partial products for $14_{five} \times 23_{five}$ are: 3, 4

$\_\_\_\__{five} \times \_\_\_\__{five}$ 3, 10

$\_\_\_\__{five} \times \_\_\_\__{five}$ 20, 4

$\_\_\_\__{five} \times \_\_\_\__{five}$ 20, 10

$\_\_\_\__{five} \times \_\_\_\__{five}$

117. Complete the blanks below to find the answer to $24_{five} \times 13_{five}$: 22

$3_{five} \times 4_{five} = \_\_\_\__{five}$ 110

$3_{five} \times 20_{five} = \_\_\_\__{five}$ 40

$10_{five} \times 4_{five} = \_\_\_\__{five}$ 200

$10_{five} \times 20_{five} = \_\_\_\__{five}$ 422

The sum of the partial products is $\_\_\_\__{five}$. Thus 422

$24_{five} \times 13_{five} = \_\_\_\__{five}$.

118. Sometimes multiplication problems are written this way:

$$\begin{array}{r} 31_{five} \\ \times 23_{five} \\ \hline \end{array}$$

$3_{five} \times \_\_\_\__{five} = 3_{five}$ 1

$3_{five} \times \_\_\_\__{five} = 140_{five}$ 30

$20_{five} \times \_\_\_\__{five} = 20_{five}$ 1

$20_{five} \times \_\_\_\__{five} = 1100_{five}$ 30

The sum of the partial products is $\_\_\_\__{five}$. Thus 1313

$31_{five} \times 23_{five} = \_\_\_\__{five}$. 1313

119. Here's an example in base ten:

$$\begin{array}{r} 23_{ten} \\ \times 39_{ten} \\ \hline \end{array}$$

$9_{ten} \times \_\_\_\__{ten} = 27_{ten}$ 3

$9_{ten} \times \_\_\_\__{ten} = 180_{ten}$ 20

$30_{ten} \times \_\_\_\__{ten} = 90_{ten}$ 3

$30_{ten} \times \_\_\_\__{ten} = 600_{ten}$ 20

The sum of the partial products is $\_\_\_\__{ten}$, and then 897

$23_{ten} \times 39_{ten} = \_\_\_\__{ten}$. 897

120. Now try this example in base ten:

$$\begin{array}{r} 36_{\text{ten}} \\ \times 21_{\text{ten}} \\ \hline \end{array}$$

$\_\_\_\__{\text{ten}} \times \_\_\_\__{\text{ten}} = 6_{\text{ten}}$ — 1, 6

$\_\_\_\__{\text{ten}} \times \_\_\_\__{\text{ten}} = 30_{\text{ten}}$ — 1, 30

$\_\_\_\__{\text{ten}} \times \_\_\_\__{\text{ten}} = 120_{\text{ten}}$ — 20, 6

$\_\_\_\__{\text{ten}} \times \_\_\_\__{\text{ten}} = 600_{\text{ten}}$ — 20, 30

Thus, $36_{\text{ten}} \times 21_{\text{ten}} = \_\_\_\__{\text{ten}}$. — 756

121. Let's try one in base six:

$$\begin{array}{r} 23_{\text{six}} \\ \times 41_{\text{six}} \\ \hline \end{array}$$

$\_\_\_\__{\text{six}} \times \_\_\_\__{\text{six}} = 3_{\text{six}}$ — 1, 3

$\_\_\_\__{\text{six}} \times \_\_\_\__{\text{six}} = 20_{\text{six}}$ — 1, 20

$\_\_\_\__{\text{six}} \times \_\_\_\__{\text{six}} = 200_{\text{six}}$ — 40, 3

$\_\_\_\__{\text{six}} \times \_\_\_\__{\text{six}} = 1200_{\text{six}}$ — 40, 20

By adding the partial products, you see that

$23_{\text{six}} \times 41_{\text{six}} = \_\_\_\__{\text{six}}$. — 1423

122. Now it's your turn to find the partial products:

$$\begin{array}{r} 24_{\text{five}} \\ \times 14_{\text{five}} \\ \hline \end{array}$$

$\_\_\_\__{\text{five}}$ — 31

$\_\_\_\__{\text{five}}$ — 130

$\_\_\_\__{\text{five}}$ — 40

$\_\_\_\__{\text{five}}$ — 200

$\_\_\_\__{\text{five}}$ *answer* — 1001

123. Multiply by finding partial products:

$$\begin{array}{r} 43_{\text{five}} \\ \times 32_{\text{five}} \\ \hline \end{array}$$

_______ five — 11

_______ five — 130

_______ five — 140

_______ five — 2200

_______ five — 3031

answer

124. In these problems we're going to leave space for you to find the partial products and the sum of the partial products:

$$\begin{array}{r} 31_{\text{five}} \\ \times 24_{\text{five}} \\ \hline \end{array}$$

_______ five — 1344

answer

125.

$$\begin{array}{r} 25_{\text{six}} \\ \times 41_{\text{six}} \\ \hline \end{array}$$

_______ six — 1545

answer

126.

$$\begin{array}{r} 39_{\text{ten}} \\ \times 48_{\text{ten}} \\ \hline \end{array}$$

_______ ten — 1872

answer

127. You may multiply 124_{five} by 32_{five} as follows:

$$\begin{array}{r} 124_{five} \\ \times 32_{five} \\ \hline \end{array}$$

$2_{five} \times 4_{five} =$ ______ $_{five}$ 13

$2_{five} \times 20_{five} =$ ______ $_{five}$ 40

$2_{five} \times 100_{five} =$ ______ $_{five}$ 200

$30_{five} \times 4_{five} =$ ______ $_{five}$ 220

$30_{five} \times 20_{five} =$ ______ $_{five}$ 1100

$30_{five} \times 100_{five} =$ ______ $_{five}$ 3000

The sum of these partial products is ______ $_{five}$. 10123

128. Write the partial products for:

$$\begin{array}{r} 133_{five} \\ \times 32_{five} \\ \hline \end{array}$$

______ $_{five}$ 11

______ $_{five}$ 110

______ $_{five}$ 200

______ $_{five}$ 140

______ $_{five}$ 1400

______ $_{five}$ 3000

______ $_{five}$ 10411

answer

129. Find the product of:

$$\begin{array}{r} 143_{ten} \\ \times 34_{ten} \\ \hline \end{array}$$

______

______ $_{ten}$ 4862

answer

Try Your Hand

These questions will help you to review and extend what you learned in this chapter. The answers to these questions appear on page 170.

1. Imagine that you are living in a strange land called *Fiveland.* All number work in Fiveland is done in base five. Tell how many fivens (Fiveland currency) you would have to spend to buy:

   a. 12_{five} grundles (Each grundle costs 3_{five} fivens.)

   b. 20_{five} mangrads (Each mangrad costs 2_{five} fivens.)

   c. 22_{five} badrafs (Each badraf costs 10_{five} fivens.)

   d. 31_{five} likmugs (Each likmug costs 20_{five} fivens.)

   e. 103_{five} snagrugs (Each snagrug costs 2_{five} fivens.)

2. Using the method of partial products, find the answer to each of the following:

   a. $3_{\text{six}} \times 12_{\text{six}}$

   b. $5_{\text{eight}} \times 31_{\text{eight}}$

   c. $2_{\text{three}} \times 22_{\text{three}}$

   d. $9_{\text{ten}} \times 18_{\text{ten}}$

   e. $20_{\text{five}} \times 14_{\text{five}}$

   f. $300_{\text{six}} \times 33_{\text{six}}$

   g. $24_{\text{nine}} \times 23_{\text{nine}}$

   h. $1004_{\text{seven}} \times 3_{\text{seven}}$

   i. $45_{\text{six}} \times 21_{\text{six}}$

   j. $35_{\text{ten}} \times 42_{\text{ten}}$

3. Using the method of partial products, try doing the following base ten multiplications *mentally:*

   a. $8_{\text{ten}} \times 13_{\text{ten}}$

   b. $7_{\text{ten}} \times 23_{\text{ten}}$

   c. $9_{\text{ten}} \times 107_{\text{ten}}$

   d. $5_{\text{ten}} \times 64_{\text{ten}}$

   e. $6_{\text{ten}} \times 220_{\text{ten}}$

   f. $9_{\text{ten}} \times 41_{\text{ten}}$

   g. $11_{\text{ten}} \times 42_{\text{ten}}$

   h. $12_{\text{ten}} \times 26_{\text{ten}}$

   i. $50_{\text{ten}} \times 35_{\text{ten}}$

   j. $34_{\text{ten}} \times 11_{\text{ten}}$

4. There are many ways of multiplying two numbers. Here's a method which is somewhat different from the one described in this chapter:

To multiply 45_{ten} by 98_{ten}, we multiply 45_{ten} by 100_{ten} and 45_{ten} by 2_{ten}. The two partial products are 4500_{ten} and 90_{ten}. Subtracting 90_{ten} from 4500_{ten}, we find that $45_{\text{ten}} \times 98_{\text{ten}} = 4410_{\text{ten}}$.

To multiply 75_{ten} by 29_{ten}, we can multiply 75_{ten} by 30_{ten} and 75_{ten} by 1_{ten}. The two partial products are 2250_{ten} and 75_{ten}. Subtracting 75_{ten} from 2250_{ten}, we find that $75_{\text{ten}} \times 29_{\text{ten}} = 2175_{\text{ten}}$.

Use this method to find the answers to the following multiplication problems:

a. $66_{\text{ten}} \times 99_{\text{ten}}$
b. $24_{\text{ten}} \times 17_{\text{ten}}$
c. $77_{\text{ten}} \times 9_{\text{ten}}$
d. $35_{\text{ten}} \times 19_{\text{ten}}$
e. $22_{\text{ten}} \times 47_{\text{ten}}$
f. $33_{\text{ten}} \times 29_{\text{ten}}$
g. $54_{\text{ten}} \times 19_{\text{ten}}$
h. $47_{\text{ten}} \times 998_{\text{ten}}$
i. $64_{\text{ten}} \times 49_{\text{ten}}$
j. $88_{\text{ten}} \times 9998_{\text{ten}}$

5. Find each of the following products:

a. $3_{\text{six}} \times 3_{\text{six}}$
b. $3_{\text{seven}} \times 3_{\text{seven}}$
c. $3_{\text{eight}} \times 3_{\text{eight}}$
d. $3_{\text{nine}} \times 3_{\text{nine}}$
e. $3_{\text{ten}} \times 3_{\text{ten}}$
f. $2_{\text{six}} \times 5_{\text{six}}$
g. $2_{\text{seven}} \times 5_{\text{seven}}$
h. $2_{\text{eight}} \times 5_{\text{eight}}$
i. $2_{\text{nine}} \times 5_{\text{nine}}$
j. $2_{\text{ten}} \times 5_{\text{ten}}$

*6. Here are five multiplication problems already done for you. Find out what base was used in each problem.

a. $4_{?} \times 5_{?} = 24_{?}$
b. $3_{?} \times 7_{?} = 23_{?}$
c. $4_{?} \times 3_{?} = 22_{?}$
d. $20_{?} \times 20_{?} = 1000_{?}$
e. $30_{?} \times 50_{?} = 2100_{?}$

*7. Here are some multiplication problems already done for you. The base for each problem is not indicated. As an additional complication, some of the answers given are incorrect—the problem just does not "fit" any base. For each problem (where possible) tell what base was used.

a. $10_{?} \times 6_{?} = 70_{?}$
b. $3_{?} \times 7_{?} = 17_{?}$
c. $4_{?} \times 3_{?} = 20_{?}$
d. $8_{?} \times 5_{?} = 30_{?}$
e. $2_{?} \times 4_{?} = 13_{?}$
f. $6_{?} \times 7_{?} = 40_{?}$
g. $4_{?} \times 8_{?} = 41_{?}$
h. $6_{?} \times 8_{?} = 30_{?}$
i. $7_{?} \times 9_{?} = 30_{?}$
j. $2_{?} \times 9_{?} = 15_{?}$

*8. Find the missing part in each of the multiplication problems below.

a. $5_{\text{six}} \times ?_{\text{six}} = 32_{\text{six}}$
b. $6_{\text{nine}} \times ?_{\text{nine}} = 46_{\text{nine}}$
c. $?_{\text{three}} \times 20_{\text{three}} = 110_{\text{three}}$
d. $5_{\text{seven}} \times ?_{\text{seven}} = 34_{\text{seven}}$
e. $30_{\text{five}} \times ?_{\text{five}} = 1400_{\text{five}}$
f. $?_{\text{six}} \times 3_{\text{six}} = 10_{\text{six}}$
g. $8_{\text{nine}} \times ?_{\text{nine}} = 35_{\text{nine}}$
h. $5_{\text{ten}} \times ?_{\text{ten}} = 75_{\text{ten}}$
i. $16_{\text{seven}} \times ?_{\text{seven}} = 54_{\text{seven}}$
j. $?_{\text{four}} \times 12_{\text{four}} = 1002_{\text{four}}$

*9. To perform the base nine multiplication $8_{\text{nine}} \times 4_{\text{nine}}$, you can do the problem in base *ten* and then "adjust" the answer to base nine. Since $8_{\text{ten}} \times 4_{\text{ten}} = 32_{\text{ten}}$, since ten is 1 more than nine, and since there are 3 tens in 32_{ten}, the answer to the base nine problem is 3 more than the answer to the base ten problem. That is, $8_{\text{ten}} \times 4_{\text{ten}} = 32_{\text{ten}}$, and $8_{\text{nine}} \times 4_{\text{nine}} = 35_{\text{nine}}$.

Use this method to find answers to the following multiplication problems:

a. $3_{\text{nine}} \times 5_{\text{nine}}$
b. $3_{\text{eight}} \times 5_{\text{eight}}$
c. $3_{\text{eight}} \times 7_{\text{eight}}$
d. $6_{\text{nine}} \times 7_{\text{nine}}$
e. $4_{\text{eight}} \times 5_{\text{eight}}$

*10. Here is another method for multiplying in base ten. This method is sometimes called the "Russian peasant" method. To find the product of 15_{ten} and 8_{ten}, you can write:

$$\begin{array}{r} \mathbf{15}_{\text{ten}} \times \mathbf{8}_{\text{ten}} \\ \mathbf{30}_{\text{ten}} \times \mathbf{4}_{\text{ten}} \\ \mathbf{60}_{\text{ten}} \times \mathbf{2}_{\text{ten}} \\ \mathbf{120}_{\text{ten}} \times \mathbf{1}_{\text{ten}} \end{array}$$

Thus we have "halved and doubled" until we found that $15_{\text{ten}} \times 8_{\text{ten}} = 120_{\text{ten}}$.

Use this method to find an answer to each of the following base ten problems.

a. $13_{\text{ten}} \times 16_{\text{ten}}$
b. $9_{\text{ten}} \times 24_{\text{ten}}$
c. $35_{\text{ten}} \times 16_{\text{ten}}$
d. $22_{\text{ten}} \times 8_{\text{ten}}$
e. $45_{\text{ten}} \times 21_{\text{ten}}$

THINK IT OVER

1. How is multiplication related to addition?

2. Here are two methods of multiplying in base ten.

$$\begin{array}{r} 24_{\text{ten}} \\ \times 16_{\text{ten}} \\ \hline 24_{\text{ten}} \\ 120_{\text{ten}} \\ 40_{\text{ten}} \\ 200_{\text{ten}} \\ \hline 384_{\text{ten}} \end{array} \qquad \begin{array}{r} 24_{\text{ten}} \\ \times 16_{\text{ten}} \\ \hline 144_{\text{ten}} \\ 240_{\text{ten}} \\ \hline 384_{\text{ten}} \end{array}$$

   How are the two methods related?

3. Some people write multiplication problems like this:

$$\begin{array}{r} 21_{\text{ten}} \\ \times 65_{\text{ten}} \\ \hline 105_{\text{ten}} \\ 126_{\text{ten}} \\ \hline 1365_{\text{ten}} \end{array}$$

   Why is the second partial product "indented" at the right?

4. Why does $10 \times 10 = 100$, regardless of what base is used?

5. In problem 4 of the Try Your Hand section of this chapter, you use a "new" method of multiplying. Can you find still other methods?

6. Why is it that some of the parts of problem 7 of the Try Your Hand section are impossible?

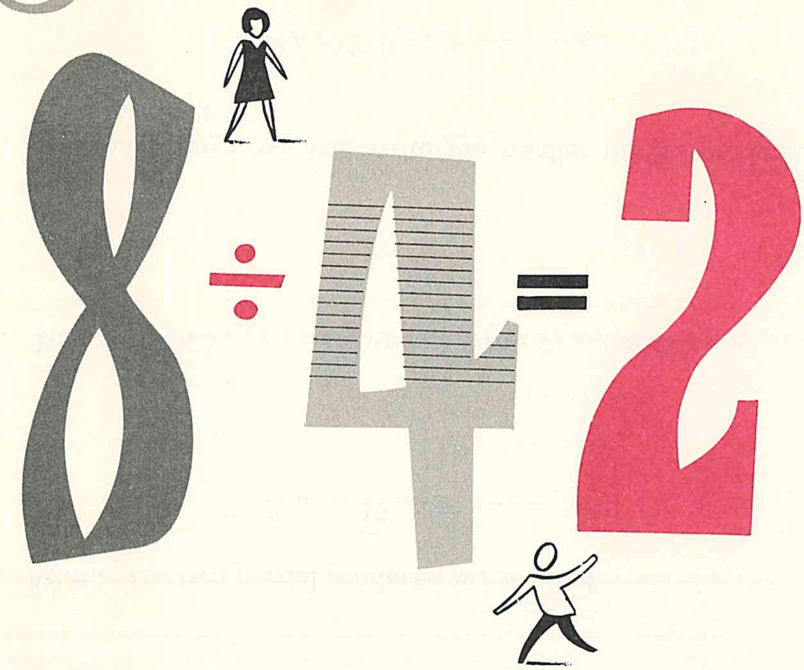
7. When you multiply 23_{five} by 14_{five}, you have four partial products. When you multiply 216_{eight} by 35_{eight}, you have six partial products. How can you predict the number of partial products you'll have in a given problem?

*8. In base ten, the **complement** of 8_{ten} is 2_{ten}. How can you find the product of 4_{ten} and 8_{ten} using the complement of 8_{ten}? Can this method be used with other bases?

*9. In base ten, $4_{\text{ten}} \times 2_{\text{ten}} = 8_{\text{ten}}$. Also, $5_{\text{ten}} \times 3_{\text{ten}} = 15_{\text{ten}}$. In base ten, the product of two one-digit numbers must be either a one-digit number or a two-digit number. Is this true for other bases? How can you predict the number of digits in the product of a given multiplication problem?

*10. In base ten, $9_{\text{ten}} + 9_{\text{ten}} = \mathbf{18}_{\text{ten}}$ and $9_{\text{ten}} \times 9_{\text{ten}} = \mathbf{81}_{\text{ten}}$. In base nine, $8_{\text{nine}} + 8_{\text{nine}} = \mathbf{17}_{\text{nine}}$ and $8_{\text{nine}} \times 8_{\text{nine}} = \mathbf{71}_{\text{nine}}$. In base eight, $7_{\text{eight}} + 7_{\text{eight}} = \mathbf{16}_{\text{eight}}$ and $7_{\text{eight}} \times 7_{\text{eight}} = \mathbf{61}_{\text{eight}}$. What pattern exists here? Does this pattern hold for other bases?

5 A Look at Division



TURNING DIVISION AROUND

■ From your knowledge of base ten, you know that **$8 \div 4 = 2$.** Can you use the numbers 2, 4, and 8 to make a statement about multiplication? Certainly you can. You can make two statements. You can say that **$2 \times 4 = 8$** and also **$4 \times 2 = 8$.**

In this chapter you are going to learn about *division* in other bases. First, you'll see that division is closely related to *multiplication.* Then you'll see that division is also related to *subtraction.* As you work through this chapter, you'll begin to see your "old" method of division in a brand new light.

Division and Multiplication

1. We can use our knowledge of multiplication to help us solve division problems such as $11_{\text{seven}} \div 2_{\text{seven}} = ?_{\text{seven}}$.

First we rewrite $11_{\text{seven}} \div 2_{\text{seven}} = ?_{\text{seven}}$ as $2_{\text{seven}} \times ?_{\text{seven}} = 11_{\text{seven}}$. Our new problem is a problem in —?—.

a. addition **b.** subtraction **c.** multiplication

c. multiplication

2. If you are asked to solve the division problem $15_{\text{seven}} \div 4_{\text{seven}} = ?_{\text{seven}}$ you could rewrite the problem as —?—.

a. $15_{\text{seven}} \times 4_{\text{seven}} = ?_{\text{seven}}$
b. $4_{\text{seven}} \times ?_{\text{seven}} = 15_{\text{seven}}$
c. $4_{\text{seven}} \div 15_{\text{seven}} = ?_{\text{seven}}$

b. $4_{\text{seven}} \times ?_{\text{seven}} = 15_{\text{seven}}$

3. Instead of solving the division problem $18_{\text{ten}} \div 6_{\text{ten}} = ?_{\text{ten}}$, you could use your knowledge of multiplication to find the answer to ——?——.

a. $6_{\text{ten}} \times ?_{\text{ten}} = 18_{\text{ten}}$

b. $6_{\text{ten}} \div ?_{\text{ten}} = 18_{\text{ten}}$

c. $18_{\text{ten}} \times ?_{\text{ten}} = 6_{\text{ten}}$

a. $6_{\text{ten}} \times ?_{\text{ten}} = 18_{\text{ten}}$

4. The division problem $121_{\text{three}} \div 2_{\text{three}} = ?_{\text{three}}$ may be rewritten as $\_\_\_\__{\text{three}} \times ?_{\text{three}} = \_\_\_\_\_\__{\text{three}}$.

2, 121

5. In base five, $100_{\text{five}} \div 10_{\text{five}} = ?_{\text{five}}$ may be written as

$\_\_\_\_\__{\text{five}} \times ?_{\text{five}} = \_\_\_\_\_\__{\text{five}}$.

10, 100

6. Rewrite $212_{\text{six}} \div 2_{\text{six}} = ?_{\text{six}}$ as a multiplication problem:

$\_\_\_\_\_\__{\text{six}} \times \_\_\_\_\_\__{\text{six}} = \_\_\_\_\_\__{\text{six}}$

2, ?, 212

7. Rewrite $1313_{\text{five}} \div 13_{\text{five}} = ?_{\text{five}}$ as a multiplication problem:

$\_\_\_\_\_\__{\text{five}} \times \_\_\_\_\_\__{\text{five}} = \_\_\_\_\_\__{\text{five}}$

13, ?, 1313

8. Here are some multiplication facts in base five:

$2_{\text{five}} \times 1_{\text{five}} = 2_{\text{five}}$	$3_{\text{five}} \times 1_{\text{five}} = 3_{\text{five}}$	$4_{\text{five}} \times 1_{\text{five}} = 4_{\text{five}}$
$2_{\text{five}} \times 2_{\text{five}} = 4_{\text{five}}$	$3_{\text{five}} \times 2_{\text{five}} = 11_{\text{five}}$	$4_{\text{five}} \times 2_{\text{five}} = 13_{\text{five}}$
$2_{\text{five}} \times 3_{\text{five}} = 11_{\text{five}}$	$3_{\text{five}} \times 3_{\text{five}} = 14_{\text{five}}$	$4_{\text{five}} \times 3_{\text{five}} = 22_{\text{five}}$
$2_{\text{five}} \times 4_{\text{five}} = 13_{\text{five}}$	$3_{\text{five}} \times 4_{\text{five}} = 22_{\text{five}}$	$4_{\text{five}} \times 4_{\text{five}} = 31_{\text{five}}$

You can use this list of multiplication facts to solve *division* problems in base five.

For example, suppose you want to solve $22_{\text{five}} \div 4_{\text{five}} = ?_{\text{five}}$. You know that you can rewrite this problem as ——?——.

a. $4_{\text{five}} \times 22_{\text{five}} = ?_{\text{five}}$

b. $4_{\text{five}} \times ?_{\text{five}} = 22_{\text{five}}$

c. $4_{\text{five}} \div 22_{\text{five}} = ?_{\text{five}}$

b. $4_{\text{five}} \times ?_{\text{five}} = 22_{\text{five}}$

9. In the list of multiplication facts, you see that $4_{five} \times 3_{five} = 22_{five}$.

Then you know that $22_{five} \div 4_{five} =$ ____ $_{five}$. 3

10. You can easily find that $14_{five} \div 3_{five} = 3_{five}$ because you know from the list of multiplication facts that

____ $_{five} \times$ ____ $_{five} =$ ____ $_{five}$. 3, 3, 14

11. Since $2_{five} \times 4_{five} = 13_{five}$, you know that $13_{five} \div 2_{five} =$ ____ $_{five}$. 4

12. In base five, $11_{five} \div 3_{five} =$ ____ $_{five}$. 2

13. And $31_{five} \div 4_{five} =$ ____ $_{five}$. 4

14. Here are some division problems in base five. Find the answer to each one:

1. $22_{five} \div 3_{five} =$ ____ $_{five}$ 4
2. $4_{five} \div 4_{five} =$ ____ $_{five}$ 1
3. $4_{five} \div 2_{five} =$ ____ $_{five}$ 2
4. $11_{five} \div 2_{five} =$ ____ $_{five}$ 3
5. $4_{five} \div 1_{five} =$ ____ $_{five}$ 4
6. $22_{five} \div 4_{five} =$ ____ $_{five}$ 3
7. $13_{five} \div 4_{five} =$ ____ $_{five}$ 2
8. $2_{five} \div 1_{five} =$ ____ $_{five}$ 2
9. $3_{five} \div 3_{five} =$ ____ $_{five}$ 1
10. $11_{five} \div 2_{five} =$ ____ $_{five}$ 3

15. Now here are some multiplication facts in base seven:

$2_{\text{seven}} \times 1_{\text{seven}} = 2_{\text{seven}}$	$3_{\text{seven}} \times 1_{\text{seven}} = 3_{\text{seven}}$
$2_{\text{seven}} \times 2_{\text{seven}} = 4_{\text{seven}}$	$3_{\text{seven}} \times 2_{\text{seven}} = 6_{\text{seven}}$
$2_{\text{seven}} \times 3_{\text{seven}} = 6_{\text{seven}}$	$3_{\text{seven}} \times 3_{\text{seven}} = 12_{\text{seven}}$
$2_{\text{seven}} \times 4_{\text{seven}} = 11_{\text{seven}}$	$3_{\text{seven}} \times 4_{\text{seven}} = 15_{\text{seven}}$
$2_{\text{seven}} \times 5_{\text{seven}} = 13_{\text{seven}}$	$3_{\text{seven}} \times 5_{\text{seven}} = 21_{\text{seven}}$
$2_{\text{seven}} \times 6_{\text{seven}} = 15_{\text{seven}}$	$3_{\text{seven}} \times 6_{\text{seven}} = 24_{\text{seven}}$

$4_{\text{seven}} \times 1_{\text{seven}} = 4_{\text{seven}}$	$5_{\text{seven}} \times 1_{\text{seven}} = 5_{\text{seven}}$
$4_{\text{seven}} \times 2_{\text{seven}} = 11_{\text{seven}}$	$5_{\text{seven}} \times 2_{\text{seven}} = 13_{\text{seven}}$
$4_{\text{seven}} \times 3_{\text{seven}} = 15_{\text{seven}}$	$5_{\text{seven}} \times 3_{\text{seven}} = 21_{\text{seven}}$
$4_{\text{seven}} \times 4_{\text{seven}} = 22_{\text{seven}}$	$5_{\text{seven}} \times 4_{\text{seven}} = 26_{\text{seven}}$
$4_{\text{seven}} \times 5_{\text{seven}} = 26_{\text{seven}}$	$5_{\text{seven}} \times 5_{\text{seven}} = 34_{\text{seven}}$
$4_{\text{seven}} \times 6_{\text{seven}} = 33_{\text{seven}}$	$5_{\text{seven}} \times 6_{\text{seven}} = 42_{\text{seven}}$

$6_{\text{seven}} \times 1_{\text{seven}} = 6_{\text{seven}}$
$6_{\text{seven}} \times 2_{\text{seven}} = 15_{\text{seven}}$
$6_{\text{seven}} \times 3_{\text{seven}} = 24_{\text{seven}}$
$6_{\text{seven}} \times 4_{\text{seven}} = 33_{\text{seven}}$
$6_{\text{seven}} \times 5_{\text{seven}} = 42_{\text{seven}}$
$6_{\text{seven}} \times 6_{\text{seven}} = 51_{\text{seven}}$

Since $3_{\text{seven}} \times 4_{\text{seven}} = 15_{\text{seven}}$, you know that

$15_{\text{seven}} \div 3_{\text{seven}} =$ ____ $_{\text{seven}}$. 4

16. In base seven, $33_{\text{seven}} \div 4_{\text{seven}} =$ ____ $_{\text{seven}}$. 6

17. And $42_{\text{seven}} \div 5_{\text{seven}} =$ ____ $_{\text{seven}}$. 6

18. Here are some division problems in base seven. Use the list of multiplication facts to help you find the answer to each one.

1. $11_{\text{seven}} \div 2_{\text{seven}} = \_\_\_\__{\text{seven}}$ 4

2. $24_{\text{seven}} \div 3_{\text{seven}} = \_\_\_\__{\text{seven}}$ 6

3. $15_{\text{seven}} \div 2_{\text{seven}} = \_\_\_\__{\text{seven}}$ 6

4. $4_{\text{seven}} \div 1_{\text{seven}} = \_\_\_\__{\text{seven}}$ 4

5. $34_{\text{seven}} \div 5_{\text{seven}} = \_\_\_\__{\text{seven}}$ 5

19. Here are some more division examples in base seven:

1. $12_{\text{seven}} \div 3_{\text{seven}} = \_\_\_\__{\text{seven}}$ 3

2. $26_{\text{seven}} \div 4_{\text{seven}} = \_\_\_\__{\text{seven}}$ 5

3. $33_{\text{seven}} \div 6_{\text{seven}} = \_\_\_\__{\text{seven}}$ 4

4. $51_{\text{seven}} \div 6_{\text{seven}} = \_\_\_\__{\text{seven}}$ 6

5. $22_{\text{seven}} \div 4_{\text{seven}} = \_\_\_\__{\text{seven}}$ 4

20. One of the multiplication facts for base seven says that

$$5_{\text{seven}} \times 3_{\text{seven}} = 21_{\text{seven}}.$$

It follows, then, that

$$5_{\text{seven}} \times 30_{\text{seven}} = \text{—?—}.$$

a. 210_{seven} b. 21_{seven} c. 150_{seven} a. 210_{seven}

21. Another multiplication fact says that

$$2_{\text{seven}} \times 4_{\text{seven}} = 11_{\text{seven}}.$$

You could say that

$$20_{\text{seven}} \times 4_{\text{seven}} = \text{—?—}.$$

a. 11_{seven} b. 110_{seven} c. 1100_{seven} b. 110_{seven}

22. Because

$$5_{\text{seven}} \times 6_{\text{seven}} = 42_{\text{seven}},$$

you can see that

$$50_{\text{seven}} \times 60_{\text{seven}} = \text{—?—}.$$

a. 42_{seven} b. 420_{seven} c. 4200_{seven} c. 4200_{seven}

23. Now let's go back to studying division. Since $40_{\text{seven}} \times 5_{\text{seven}} =$ 260_{seven}, you know that $260_{\text{seven}} \div 5_{\text{seven}} =$ ______ seven. 40

24. In base seven, $300_{\text{seven}} \times 5_{\text{seven}} = 2100_{\text{seven}}$. Then $2100_{\text{seven}} \div 5_{\text{seven}} =$ ______ seven. 300

25. In base seven, $2600_{\text{seven}} \div 5_{\text{seven}} =$ —?—.

a. 4_{seven} **b.** 40_{seven} **c.** 400_{seven} **c.** 400_{seven}

26. And $5100_{\text{seven}} \div 6_{\text{seven}} =$ —?—.

a. 6_{seven} **b.** 60_{seven} **c.** 600_{seven} **c.** 600_{seven}

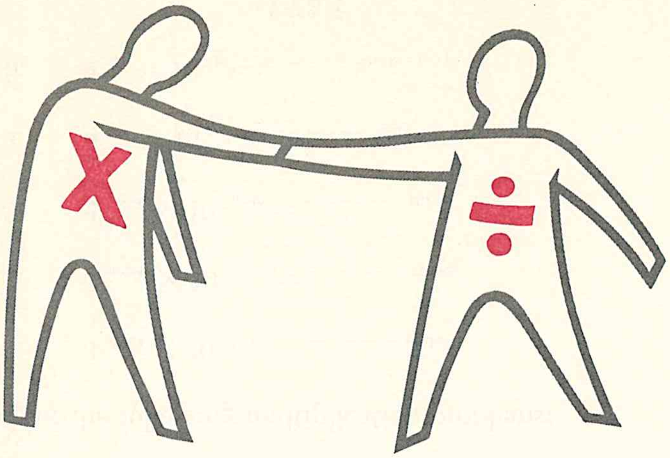
27. Try each of the following division problems:

1. $330_{\text{seven}} \div 4_{\text{seven}} =$ ______ seven 60
2. $2400_{\text{seven}} \div 6_{\text{seven}} =$ ______ seven 300
3. $13_{\text{seven}} \div 5_{\text{seven}} =$ ______ seven 2
4. $420_{\text{seven}} \div 6_{\text{seven}} =$ ______ seven 50
5. $15000_{\text{seven}} \div 6_{\text{seven}} =$ ______ seven 2000

28. Since $10_{\text{seven}} \times 5_{\text{seven}} = 50_{\text{seven}}$, you can say that $50_{\text{seven}} \div 5_{\text{seven}} =$ ______ seven. 10

29. In base seven, $100_{\text{seven}} \times 42_{\text{seven}} = 4200_{\text{seven}}$. Then $4200_{\text{seven}} \div 42_{\text{seven}} =$ ______ seven. 100

30. $32000_{\text{seven}} \div 32_{\text{seven}} =$ ______ seven. 1000



31. Since $21_{\text{seven}} \times 100_{\text{seven}} = 2100_{\text{seven}}$, $2100_{\text{seven}} \div 100_{\text{seven}} =$ —?—.

a. 21_{seven} b. 210_{seven} c. 2100_{seven}

a. 21_{seven}

32. In base seven, $56000_{\text{seven}} \div 1000_{\text{seven}} =$ ______ seven.

56

33. Since $3100_{\text{seven}} \div 31_{\text{seven}} = 100_{\text{seven}}$, $3100_{\text{seven}} \div 310_{\text{seven}} =$ —?—.

a. 10_{seven} b. 100_{seven} c. 1000_{seven}

a. 10_{seven}

34. $4200_{\text{seven}} \div 420_{\text{seven}} =$ ______ seven.

10

35. And since $3100_{\text{seven}} \div 100_{\text{seven}} = 31_{\text{seven}}$, $3100_{\text{seven}} \div 10_{\text{seven}} =$ —?—.

a. 10_{seven} b. 31_{seven} c. 310_{seven}

c. 310_{seven}

36. Complete the blanks to solve each of the following division problems:

1. $220_{\text{seven}} \div 10_{\text{seven}} =$ ______ seven — 22
2. $14000_{\text{seven}} \div 10_{\text{seven}} =$ ______ seven — 1400
3. $2400_{\text{seven}} \div 240_{\text{seven}} =$ ______ seven — 10
4. $54000_{\text{seven}} \div 540_{\text{seven}} =$ ______ seven — 100
5. $36000_{\text{seven}} \div 100_{\text{seven}} =$ ______ seven — 360

37. Now let's go back to base five. Here is the summary of multiplication facts for base five:

$2_{five} \times 1_{five} = 2_{five}$	$3_{five} \times 1_{five} = 3_{five}$	$4_{five} \times 1_{five} = 4_{five}$
$2_{five} \times 2_{five} = 4_{five}$	$3_{five} \times 2_{five} = 11_{five}$	$4_{five} \times 2_{five} = 13_{five}$
$2_{five} \times 3_{five} = 11_{five}$	$3_{five} \times 3_{five} = 14_{five}$	$4_{five} \times 3_{five} = 22_{five}$
$2_{five} \times 4_{five} = 13_{five}$	$3_{five} \times 4_{five} = 22_{five}$	$4_{five} \times 4_{five} = 31_{five}$

In base five,

$$4_{five} \times 3_{five} = 22_{five}.$$

Thus, you can see that

$$4_{five} \times 30_{five} = \underline{\hspace{3em}}_{five}.$$ 220

38. Then, $220_{five} \div 30_{five} = \underline{\hspace{2em}}_{five}$. 4

39. Since $3_{five} \times 20_{five} = 110_{five}$, $110_{five} \div 20_{five} = \underline{\hspace{2em}}_{five}$. 3

40. In base five, $20_{five} \times 40_{five} = 1300_{five}$. Then

$1300_{five} \div 40_{five} = \underline{\hspace{3em}}_{five}$. 20

41. $200_{five} \times 40_{five} = \underline{\hspace{4em}}_{five}$. Then 13000

$13000_{five} \div 40_{five} = \underline{\hspace{3em}}_{five}$. 200

42. Try the following division problems in base five:

1. $2200_{five} \div 3_{five} = \underline{\hspace{3em}}_{five}$ 400
2. $110_{five} \div 20_{five} = \underline{\hspace{2em}}_{five}$ 3
3. $4000_{five} \div 400_{five} = \underline{\hspace{3em}}_{five}$ 10
4. $13000_{five} \div 400_{five} = \underline{\hspace{3em}}_{five}$ 20
5. $1400_{five} \div 30_{five} = \underline{\hspace{3em}}_{five}$ 30

43. Here are some more division problems in base five:

1. $220_{five} \div 30_{five} =$ ______ $_{five}$ — 4

2. $1400_{five} \div 30_{five} =$ ______ $_{five}$ — 30

3. $400_{five} \div 2_{five} =$ ______ $_{five}$ — 200

4. $1300_{five} \div 2_{five} =$ ______ $_{five}$ — 400

5. $11000_{five} \div 200_{five} =$ ______ $_{five}$ — 30

Another Method of Division

44. In all the division problems you've seen so far, you have been able to use a list of multiplication facts to help you find the **quotients.** (A *quotient* is the answer to a division problem.) Since you will be studying division in several different bases, you are now going to look at a method of division which does not depend on your knowledge of multiplication facts for a given base. This new method depends upon subtraction.

Let's use the new method to find the quotient for the division problem $10_{six} \div 2_{six} = ?_{six}$. Instead of using a list of multiplication facts, we're going to *subtract* 2_{six} from 10_{six}.

$$\begin{array}{r} 10_{six} \\ -2_{six} \\ \hline ? \end{array}$$

a. 2_{six} b. 4_{six} c. 8_{six} — b. 4_{six}

45. We can continue to subtract 2_{six} from 10_{six} until we reach 0_{six}:

$$\begin{array}{r} 10_{six} \\ -2_{six} \\ \hline 4_{six} \\ -2_{six} \\ \hline 2_{six} \\ -2_{six} \\ \hline 0_{six} \end{array}$$

We were able to subtract 2_{six} from 10_{six} ——?—— time(s) before we finally reached 0_{six}.

a. three b. four c. ten — a. three

46. Then we know that $10_{six} \div 2_{six} =$ ——?——.

a. 3_{six} b. 4_{six} c. 10_{six} — a. 3_{six}

47. Now let's try a problem in base five. Let's divide 13_{five} by 4_{five}:

$$\begin{array}{r} 13_{five} \\ -4_{five} \\ \hline \\ \_\_\_\__{five} \\ -4_{five} \\ \hline 0_{five} \end{array}$$

4

We were able to subtract 4_{five} from 13_{five} —?— times before we finally reached 0_{five}.

a. two **b.** three **c.** four

a. two

48. Then $13_{five} \div 4_{five} = \_\_\_\__{five}$.

2

49. Let's try a problem in base eight. Let's find the quotient for $10_{eight} \div 2_{eight}$:

$$\begin{array}{r} 10_{eight} \\ -2_{eight} \\ \hline \\ \_\_\_\__{eight} \\ -2_{eight} \\ \hline \\ \_\_\_\__{eight} \\ -2_{eight} \\ \hline \\ \_\_\_\__{eight} \\ -2_{eight} \\ \hline \\ \_\_\_\__{eight} \end{array}$$

6

4

2

0

We were able to subtract 2_{eight} from 10_{eight} —?— time(s) before we reached 0_{eight}.

a. one **b.** four **c.** ten

b. four

50. Then $10_{\text{eight}} \div 2_{\text{eight}} = \_\_\_\_\__{\text{eight}}$. 4

51. Let's try a division problem in base nine. Let's divide 20_{nine} by 6_{nine}:

$$\begin{array}{r} 20_{\text{nine}} \\ -6_{\text{nine}} \\ \hline \_\_\_\__{\text{nine}} \\ -6_{\text{nine}} \\ \hline \_\_\_\__{\text{nine}} \\ -6_{\text{nine}} \\ \hline \_\_\_\__{\text{nine}} \end{array}$$

13

6

0

Thus you see that $20_{\text{nine}} \div 6_{\text{nine}} = \_\_\_\_\__{\text{nine}}$. 3

52. Now, in base seven, let's use the subtraction method to divide 21_{seven} by 5_{seven}:

$$\begin{array}{r} 21_{\text{seven}} \\ -5_{\text{seven}} \\ \hline \_\_\_\__{\text{seven}} \\ -5_{\text{seven}} \\ \hline \_\_\_\__{\text{seven}} \\ -5_{\text{seven}} \\ \hline \_\_\_\__{\text{seven}} \end{array}$$

13

5

0

The quotient for $21_{\text{seven}} \div 5_{\text{seven}}$ is $\_\_\_\_\__{\text{seven}}$. 3

53. Here's one to do all by yourself. In base three, use the subtraction method to divide 11_{three} by 2_{three}:

You can see that $11_{\text{three}} \div 2_{\text{three}} = \_\_\_\_\__{\text{three}}$. 2

54. Here are some division problems for you to solve. Use the subtraction method to find the quotient for each one. Do the computation work on scrap paper.

1. $22_{\text{five}} \div 4_{\text{five}} = \_\_\_\__{\text{five}}$ 3

2. $12_{\text{four}} \div 3_{\text{four}} = \_\_\_\__{\text{four}}$ 2

3. $17_{\text{eight}} \div 5_{\text{eight}} = \_\_\_\__{\text{eight}}$ 3

4. $20_{\text{six}} \div 4_{\text{six}} = \_\_\_\__{\text{six}}$ 3

55. If a list of multiplication facts in the various bases had been available, you could have solved all the previous division problems directly from the table. The subtraction method, however, can be used even when a list of multiplication facts is *not* available.

Let's look at a division problem for which a list of multiplication facts would be hard to find. Let's divide 54_{seven} by 16_{seven}:

$$
\begin{array}{r}
54_{\text{seven}} \\
\underline{-16_{\text{seven}}} \\
\_\_\_\__{\text{seven}} \\
\underline{-16_{\text{seven}}} \\
\_\_\_\__{\text{seven}} \\
\underline{-16_{\text{seven}}} \\
\_\_\_\__{\text{seven}}
\end{array}
$$

35

16

0

Since we were able to subtract 16_{seven} three times from 54_{seven}, we know that $54_{\text{seven}} \div 16_{\text{seven}} = \_\_\_\__{\text{seven}}$. 3

56. We can, of course, use the old, familiar method of writing a division problem. The subtraction method is the method we'll use. Try this one:

$$
\begin{array}{r}
12_{\text{five}}\,\overline{)\,41_{\text{five}}} \\
\underline{-12_{\text{five}}} \\
\_\_\_\__{\text{five}} \\
\underline{-12_{\text{five}}} \\
\_\_\_\__{\text{five}} \\
\underline{-12_{\text{five}}} \\
\_\_\_\__{\text{five}}
\end{array}
$$

24

12

0

The quotient for $41_{\text{five}} \div 12_{\text{five}}$ is $\_\_\_\__{\text{five}}$. 3

57. Now let's try a problem in base six:

$$
\begin{array}{r l}
34_{\text{six}}) & \overline{112_{\text{six}}} \\
 & \underline{-34_{\text{six}}} \\
 & \_\_\_\__{\text{six}} \\
 & \underline{-34_{\text{six}}} \\
 & \_\_\_\__{\text{six}}
\end{array}
$$

34

0

You can see that $112_{\text{six}} \div 34_{\text{six}} = \_\_\_\__{\text{six}}$. 2

58. Try this one on your own:

$$22_{\text{three}}\overline{)\,121_{\text{three}}}$$

The quotient for $121_{\text{three}} \div 22_{\text{three}}$ is $\_\_\_\__{\text{three}}$. 2

59. Use the subtraction method to solve each of these division problems. Do your computations on scrap paper.

1. $52_{\text{eight}} \div 16_{\text{eight}} = \_\_\_\__{\text{eight}}$ 3
2. $212_{\text{five}} \div 34_{\text{five}} = \_\_\_\__{\text{five}}$ 3
3. $1112_{\text{six}} \div 334_{\text{six}} = \_\_\_\__{\text{six}}$ 2
4. $66_{\text{seven}} \div 11_{\text{seven}} = \_\_\_\__{\text{seven}}$ 6
5. $432_{\text{five}} \div 124_{\text{five}} = \_\_\_\__{\text{five}}$ 3

60. Suppose you were asked to divide 20_{four} by 2_{four} and you decided to use the subtraction method:

$$
\begin{array}{r l}
2_{\text{four}}) & \overline{20_{\text{four}}} \\
 & \underline{-2_{\text{four}}} \\
 & \_\_\_\__{\text{four}} \\
 & \underline{-2_{\text{four}}} \\
 & \_\_\_\__{\text{four}} \\
 & \underline{-2_{\text{four}}} \\
 & \_\_\_\__{\text{four}} \\
 & \underline{-2_{\text{four}}} \\
 & \_\_\_\__{\text{four}}
\end{array}
$$

12

10

2

0

Since you were able to subtract 2_{four} from 20_{four} four times, you know that $20_{\text{four}} \div 2_{\text{four}} =$ ——?——.

a. 4_{four} b. 10_{four} c. 3_{four} b. 10_{four}

61. Of course, if you had wanted to, you could have used *multiplication facts* to find the quotient of $20_{\text{four}} \div 2_{\text{four}}$. You know that $2_{\text{four}} \times 10_{\text{four}} = 20_{\text{four}}$, and thus it would have been easy to see

that $20_{\text{four}} \div 2_{\text{four}} =$ ______ $_{\text{four}}$. 10

62. Using the subtraction method, let's find the quotient of $20_{\text{three}} \div 2_{\text{three}}$:

$$
\begin{array}{r}
2_{\text{three}}\,\overline{)\,20_{\text{three}}} \\
-2_{\text{three}} \\
\hline
\_\_\_\__{\text{three}} \\
-2_{\text{three}} \\
\hline
\_\_\_\__{\text{three}} \\
-2_{\text{three}} \\
\hline
\_\_\_\__{\text{three}}
\end{array}
$$

11

2

0

We subtracted 2_{three} three times. Thus we can say that $20_{\text{three}} \div 2_{\text{three}} =$ —?—.

a. 3_{three} b. 10_{three} c. 2_{three} b. 10_{three}

Combining Steps

63. Instead of subtracting 2_{three} three times in a problem such as the one above, we can combine all the steps into one. We must be sure to keep track of the number of times we subtract 2_{three}.

$$
\begin{array}{rl}
2_{\text{three}}\,\overline{)\,20_{\text{three}}} & \\
20_{\text{three}} & \quad 10_{\text{three}} \\
\hline
0_{\text{three}} &
\end{array}
$$

The 10_{three} tells us that we subtracted 2_{three} —?— times in one step.

a. two b. three c. four b. three

64. Let's divide 30_{five} by 3_{five}:

$$
\begin{array}{rl}
3_{\text{five}}\,\overline{)\,30_{\text{five}}} & \\
30_{\text{five}} & \quad 10_{\text{five}} \\
\hline
0_{\text{five}} &
\end{array}
$$

Thus, in base five, $30_{\text{five}} \div 3_{\text{five}} =$ ______ $_{\text{five}}$. 10

65. Suppose we want to divide 22_{seven} by 2_{seven}:

$$2_{seven}\overline{)\,22_{seven}}$$

In the first step we get a **partial quotient** of 10_{seven}:

$$\begin{array}{r l}
2_{seven}\overline{)\,22_{seven}} & \\
\underline{20_{seven}} & \quad 10_{seven} \\
\underline{\qquad}_{seven} &
\end{array}$$

2

But you can see that we haven't yet reached 0_{seven}. Let's continue:

$$\begin{array}{r l}
2_{seven}\overline{)\,22_{seven}} & \\
\underline{20_{seven}} & \quad 10_{seven} \\
2_{seven} & \\
\underline{2_{seven}} & \quad 1_{seven} \\
\underline{\qquad}_{seven} &
\end{array}$$

0

The *sum* of our partial quotients, 10_{seven} and 1_{seven}, tells us our final quotient. You can see that $22_{seven} \div 2_{seven} = \underline{\qquad}_{seven}$.

11

66. Let's divide 36_{nine} by 3_{nine}. In the first step we get 10_{nine} as a partial quotient:

$$\begin{array}{r l}
3_{nine}\overline{)\,36_{nine}} & \\
\underline{30_{nine}} & \quad 10_{nine} \\
\underline{\qquad}_{nine} &
\end{array}$$

6

But you see that we're not finished yet:

$$\begin{array}{r l}
3_{nine}\overline{)\,36_{nine}} & \\
\underline{30_{nine}} & \quad 10_{nine} \\
6_{nine} & \\
\underline{3_{nine}} & \quad \underline{\qquad}_{nine} \\
\underline{\qquad}_{nine} & \\
\underline{3_{nine}} & \quad \underline{\qquad}_{nine} \\
\underline{\qquad}_{nine} &
\end{array}$$

1

3
1

0

The sum of the partial quotients, 10_{nine}, 1_{nine}, and 1_{nine}, tells us the final quotient. You can see that $36_{nine} \div 3_{nine} = \underline{\qquad}_{nine}$.

12

67. Here's a problem in base five. First we get 10_{five} as a partial quotient:

$4_{five}\overline{)\,134_{five}}$

40_{five} $\quad 10_{five}$

____ $_{five}$ 44

But we can subtract 40_{five} once again:

$4_{five}\overline{)\,134_{five}}$

40_{five} $\quad 10_{five}$

44_{five}

40_{five} $\quad$ ____ $_{five}$ 10

____ $_{five}$ 4

And finally, we subtract 4_{five} to get 0_{five}:

$4_{five}\overline{)\,134_{five}}$

40_{five} $\quad 10_{five}$

44_{five}

40_{five} $\quad 10_{five}$

4_{five}

4_{five} $\quad$ ____ $_{five}$ 1

0_{five}

The sum of the partial quotients tells you that

$134_{five} \div 4_{five} =$ ____ $_{five}$. 21

68. Try this division problem in base seven:

$5_{seven}\overline{)\,143_{seven}}$

50_{seven} $\quad 10_{seven}$

____ $_{seven}$ 63

____ $_{seven}$ $\quad 10_{seven}$ 50

13_{seven}

____ $_{seven}$ $\quad 1_{seven}$ 5

____ $_{seven}$ 5

5_{seven} $\quad 1_{seven}$

____ $_{seven}$ 0

Thus, you see that $143_{seven} \div 5_{seven} =$ ____ $_{seven}$. 22

69. Here's a division problem in base ten:

$$7_{\text{ten}}\overline{)\,147_{\text{ten}}}$$

_____ ten	10_{ten}	70
_____ ten		77
_____ ten	10_{ten}	70
_____ ten		7
_____ ten	1_{ten}	7
0_{ten}		

You can see that $147_{\text{ten}} \div 7_{\text{ten}} =$ ______ ten. 21

70. Here's a division problem for you to try on your own:

$$6_{\text{eight}}\overline{)\,226_{\text{eight}}}$$

The quotient for $226_{\text{eight}} \div 6_{\text{eight}}$ is ______ eight. 31

71. Now try these division problems. Do the computation on scrap paper.

1. $51_{\text{seven}} \div 4_{\text{seven}} =$ ______ seven 12
2. $165_{\text{nine}} \div 7_{\text{nine}} =$ ______ nine 22
3. $124_{\text{eight}} \div 7_{\text{eight}} =$ ______ eight 14
4. $143_{\text{five}} \div 3_{\text{five}} =$ ______ five 31
5. $112_{\text{three}} \div 2_{\text{three}} =$ ______ three 21

72. We can easily extend the method we have been using. Suppose we divide 311_{five} by 3_{five}.

Our first step gives us a partial quotient of 100_{five}:

$$
\begin{array}{r|l}
3_{five}\,)\,311_{five} & \\
300_{five} & 100_{five} \\
\hline
\_\_\_\__{five} &
\end{array}
$$

11

But we still haven't reached 0_{five}. Let's continue to subtract:

$$
\begin{array}{r|l}
3_{five}\,)\,311_{five} & \\
300_{five} & 100_{five} \\
\hline
11_{five} & \\
\_\_\_\__{five} & 1_{five} \\
\hline
\_\_\_\__{five} & \\
\_\_\_\__{five} & 1_{five} \\
\hline
0_{five} &
\end{array}
$$

3

3

3

The quotient for $311_{five} \div 3_{five}$ is ______ $_{five}$. 102

73. Here's another problem similar to the last one:

$$
\begin{array}{r|l}
4_{seven}\,)\,411_{seven} & \\
400_{seven} & \_\_\_\__{seven} \\
\hline
\_\_\_\__{seven} & \\
4_{seven} & \_\_\_\__{seven} \\
\hline
\_\_\_\__{seven} & \\
4_{seven} & \_\_\_\__{seven} \\
\hline
0_{seven} &
\end{array}
$$

100

11

1

4

1

When you find the sum of the partial products, you know that

$411_{seven} \div 4_{seven} =$ ______ $_{seven}$. 102

74. In base five, try this one:

$$
\begin{array}{r|l}
4_{five}\,)\,1304_{five} & \\
400_{five} & \_\_\_\__{five} \\
\hline
404_{five} & \\
\_\_\_\__{five} & 100_{five} \\
\hline
\_\_\_\__{five} & \\
4_{five} & \_\_\_\__{five} \\
\hline
0_{five} &
\end{array}
$$

100

400

4

1

You can see that $1304_{five} \div 4_{five} =$ ______ $_{five}$. 201

75. Here's an example in base six:

$$5_{\text{six}}\overline{)\,1045_{\text{six}}}$$

$\underline{}_{\text{six}}$	100_{six}	500
145_{six}		
$\underline{50_{\text{six}}}$	$\underline{}_{\text{six}}$	10
55_{six}		
$\underline{}_{\text{six}}$	10_{six}	50
$\underline{}_{\text{six}}$		5
$\underline{5_{\text{six}}}$	$\underline{}_{\text{six}}$	1
0_{six}		

The sum of the partial quotients is $\underline{}_{\text{six}}$. 121

76. Here are some division problems in assorted bases. Try each one. Do your computations on scrap paper.

1. $615_{\text{seven}} \div 6_{\text{seven}} = \underline{}_{\text{seven}}$ 102
2. $1788_{\text{nine}} \div 8_{\text{nine}} = \underline{}_{\text{nine}}$ 211
3. $544_{\text{six}} \div 4_{\text{six}} = \underline{}_{\text{six}}$ 124
4. $3011_{\text{five}} \div 3_{\text{five}} = \underline{}_{\text{five}}$ 1002
5. $13212_{\text{four}} \div 3_{\text{four}} = \underline{}_{\text{four}}$ 2202

77. Thus far, most of the divisors you've seen have had only one digit. Let's look at some problems in which the divisor has more than one digit. For example:

$$12_{\text{five}}\overline{)\,314_{\text{five}}}$$

$\underline{}_{\text{five}}$	10_{five}	120
144_{five}		
$\underline{}_{\text{five}}$	10_{five}	120
$\underline{}_{\text{five}}$		24
$\underline{12_{\text{five}}}$	$\underline{}_{\text{five}}$	1
12_{five}		
$\underline{}_{\text{five}}$	1_{five}	12
0_{five}		

Thus, $314_{\text{five}} \div 12_{\text{five}} = \underline{}_{\text{five}}$. 22

78. In base eight, try this one:

$$37_{\text{eight}}\overline{)\,1017_{\text{eight}}}$$

$\_\_\_\__{\text{eight}}$	10_{eight}	370
427_{eight}		
370_{eight}	$\_\_\_\__{\text{eight}}$	10
37_{eight}		
37_{eight}	$\_\_\_\__{\text{eight}}$	1
0_{eight}		

You can see that $1017_{\text{eight}} \div 37_{\text{eight}} = \_\_\_\__{\text{eight}}$. 21

79. In base six, try this one:

$$204_{\text{six}}\overline{)\,4532_{\text{six}}}$$

$\_\_\_\__{\text{six}}$	10_{six}	2040
2452_{six}		
$\_\_\_\__{\text{six}}$	10_{six}	2040
$\_\_\_\__{\text{six}}$		412
204_{six}	$\_\_\_\__{\text{six}}$	1
204_{six}		
$\_\_\_\__{\text{six}}$	1_{six}	204
0_{six}		

The quotient for $4532_{\text{six}} \div 204_{\text{six}}$ is $\_\_\_\__{\text{six}}$. 22

80. In base ten, find the quotient for this problem:

$$316_{\text{ten}}\overline{)\,31916_{\text{ten}}}$$

31600_{ten}	$\_\_\_\__{\text{ten}}$	100
$\_\_\_\__{\text{ten}}$		316
$\_\_\_\__{\text{ten}}$	1_{ten}	316
0_{ten}		

Thus $31916_{\text{ten}} \div 316_{\text{ten}} = \_\_\_\__{\text{ten}}$. 101

81. Try this one on your own:

$$45_{\text{seven}} \overline{)\,6105_{\text{seven}}}$$

The quotient for $6105_{\text{seven}} \div 45_{\text{seven}}$ is ________ $_{\text{seven}}$. 121

82. Find answers to each of the following division problems. Perform the computations on scrap paper.

1. $1341_{\text{five}} \div 32_{\text{five}} =$ ________ $_{\text{five}}$ 23
2. $2022_{\text{four}} \div 113_{\text{four}} =$ ________ $_{\text{four}}$ 12
3. $2080_{\text{ten}} \div 16_{\text{ten}} =$ ________ $_{\text{ten}}$ 130
4. $11221_{\text{three}} \div 21_{\text{three}} =$ ________ $_{\text{three}}$ 201
5. $50401_{\text{six}} \div 41_{\text{six}} =$ ________ $_{\text{six}}$ 1121

Try Your Hand

■ These questions will help you review what you learned in Chapter 5. The answers appear on page 170.

1. Find quotients for the following division problems:

a. $44_{\text{eight}} \div 4_{\text{eight}}$

b. $32_{\text{six}} \div 5_{\text{six}}$

c. $62_{\text{nine}} \div 8_{\text{nine}}$

d. $100_{\text{six}} \div 4_{\text{six}}$

e. $224_{\text{nine}} \div 8_{\text{nine}}$

f. $141_{\text{seven}} \div 6_{\text{seven}}$

g. $111_{\text{four}} \div 3_{\text{four}}$

h. $813_{\text{nine}} \div 36_{\text{nine}}$

i. $2024_{\text{five}} \div 44_{\text{five}}$

j. $2030_{\text{four}} \div 130_{\text{four}}$

2. Find the missing part in each of the division problems below:

a. $12_{\text{five}} \div ?_{\text{five}} = 1_{\text{five}}$

b. $26_{\text{seven}} \div ?_{\text{seven}} = 5_{\text{seven}}$

c. $?_{\text{eight}} \div 5_{\text{eight}} = 6_{\text{eight}}$

d. $75_{\text{ten}} \div ?_{\text{ten}} = 5_{\text{ten}}$

e. $?_{\text{three}} \div 11_{\text{three}} = 2_{\text{three}}$

f. $?_{\text{eight}} \div 5_{\text{eight}} = 7_{\text{eight}}$

g. $55_{\text{seven}} \div ?_{\text{seven}} = 5_{\text{seven}}$

h. $?_{\text{three}} \div 12_{\text{three}} = 12_{\text{three}}$

i. $?_{\text{ten}} \div 26_{\text{ten}} = 5_{\text{ten}}$

j. $22_{\text{four}} \div ?_{\text{four}} = 11_{\text{four}}$

3. Here's a "new" way of looking at division. To divide 98_{ten} by 7_{ten}, we express 98_{ten} as $70_{\text{ten}} + 28_{\text{ten}}$. Then we divide both 70_{ten} and 28_{ten} by 7_{ten}. The two partial quotients, 10_{ten} and 4_{ten}, tell us that $98_{\text{ten}} \div 7_{\text{ten}} = 14_{\text{ten}}$. Use this method to find answers to the following division problems.

a. $78_{\text{ten}} \div 6_{\text{ten}}$

b. $42_{\text{seven}} \div 3_{\text{seven}}$

c. $116_{\text{nine}} \div 8_{\text{nine}}$

d. $524_{\text{eight}} \div 5_{\text{eight}}$

e. $144_{\text{ten}} \div 8_{\text{ten}}$

f. $126_{\text{nine}} \div 7_{\text{nine}}$

g. $413_{\text{five}} \div 4_{\text{five}}$

h. $204_{\text{six}} \div 2_{\text{six}}$

i. $143_{\text{ten}} \div 13_{\text{ten}}$

j. $228_{\text{ten}} \div 19_{\text{ten}}$

4. If there are 12_{ten} objects in a basket, the objects can be divided equally among three friends because $12_{\text{ten}} \div 3_{\text{ten}} = 4_{\text{ten}}$. Tell whether the following collections can be divided equally among three friends:

a. 16_{nine} badrufs

b. 24_{five} scandliks

c. 32_{six} budjeebs

d. 71_{nine} haggles

e. 12_{three} pathebs

f. 100_{seven} rattlers

g. 66_{nine} divots

h. 306_{ten} hibbards

i. 100_{nine} skillups

j. 144_{six} ballywigs

5. The following problems have been done for you. Find the base in which each problem was done.

a. $22_? \div 3_? = 4_?$

b. $15_? \div 2_? = 7_?$

c. $40_? \div 8_? = 5_?$

d. $52_? \div 4_? = 12_?$

e. $101_? \div 2_? = 12_?$

6. Here are ten division problems. Some of them "come out even" and some of them don't. (By "come out even" we mean that there is no remainder.) Tell which of the problems "come out even" and which do not.

a. $22_{\text{four}} \div 3_{\text{four}}$

b. $21_{\text{five}} \div 2_{\text{five}}$

c. $36_{\text{nine}} \div 3_{\text{nine}}$

d. $47_{\text{eight}} \div 4_{\text{eight}}$

e. $52_{\text{seven}} \div 2_{\text{seven}}$

f. $45_{\text{eight}} \div 6_{\text{eight}}$

g. $25_{\text{six}} \div 5_{\text{six}}$

h. $31_{\text{four}} \div 2_{\text{four}}$

i. $33_{\text{five}} \div 3_{\text{five}}$

j. $36_{\text{eight}} \div 6_{\text{eight}}$

THINK IT OVER

■ These questions will make you think hard about what you've learned about division. Try each one. Don't be afraid to experiment even though your answers might be different from those of your classmates.

1. How is division related to multiplication? To subtraction?

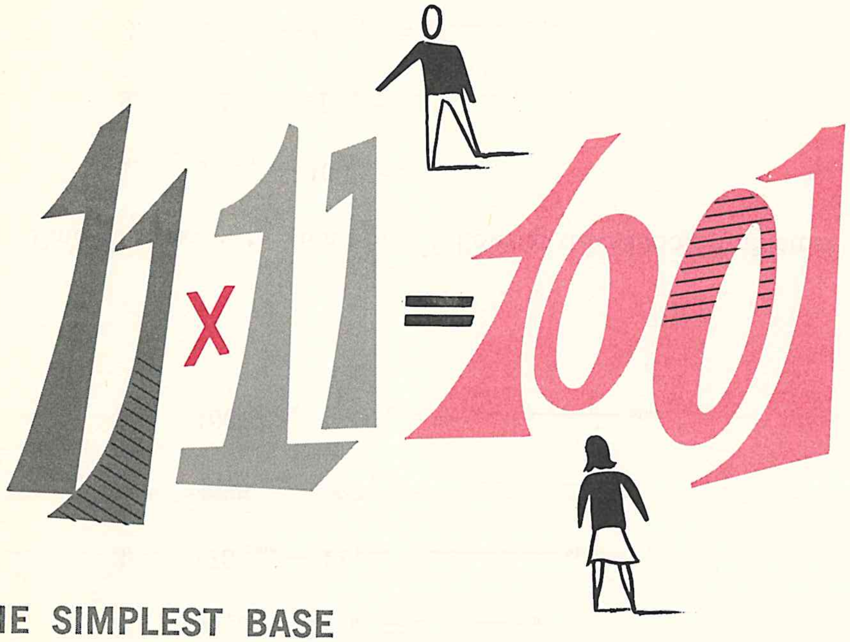
2. How is the "subtraction" method of division related to the "partial quotients" method?

3. How is the division method in problem 3 of the Try Your Hand section above related to the method you studied in this chapter?

4. In base ten, you can easily tell whether or not a number is divisible by ten. The numeral must end in 0. How can you tell when a *base five* numeral represents a number divisible by *five?* How can you tell when a *base six* numeral represents a number divisible by *six?* Does this pattern *always* hold?

5. In base ten, you can easily tell whether or not a number is divisible by 2—the numeral must end in 2, 4, 6, or 8. How can you tell whether or not a number is divisible by 3? By 4? By 6?

6 A Look at Base Two



THE SIMPLEST BASE

In this chapter you will have a chance to learn about one of the most useful, interesting, and simple bases in all of mathematics—**base two.** In base two you will see some more unusual number facts. You will see, for example, that $11_{\text{two}} \times 11_{\text{two}} = 1001_{\text{two}}$.

Aside from studying base two number facts, you will take a brief look at an application of base two in a new and vastly important field—computers.

Using Base Two Numerals

1. In base ten, you use *ten* digits. To write numerals in base five, you use —?— digits.

   a. two **b.** five **c.** ten

   b. five

2. You would expect to use —?— digits in base *two*.

   a. two **b.** five **c.** ten

   a. two

3. We're going to write all numerals in base two by using the digits 0 and 1.

   The numeral for *one* in base two is ——$_{\text{two}}$.

   1

4. The numeral 10_{two} means ——?——.

a. five b. ten c. two c. two

1	0
TWOS	ONES

5. You remember that, in base *ten*, the 1 in 10_{ten} is ________ times ten
the 1 in 1_{ten}. And the 1 in 100_{ten} is ________ times the 1 in 10_{ten}. ten

6. In base five the 1 in 10_{five} is ________ times the 1 in 1_{five}, and five
the 1 in 100_{five} is ________ times the 1 in 10_{five}. five

7. Then, in base two, the 1 in 10_{two} is ——?—— times the 1 in 1_{two}.

a. two b. five c. ten a. two

8. Thus 10_{two} means 1 ________. two

9. And 11_{two} means ______ two(s) and ______ one(s). 1, 1

10. You would write 11_{two} in base ten as _____ $_{ten}$. 3

1	1
TWOS	ONES

11. The 1 in 100_{two} is ——?—— times the 1 in 10_{two}.

a. two b. five c. ten a. two

12. Thus 100_{two} means 1 ——?——.

a. four b. twenty-five c. one hundred a. four

13. And 111_{two} means ______ four(s), ______ two(s), and ______ one(s). 1, 1, 1

14. You would write 111_{two} in base ten as $\_\_\_\__{ten}$. 7

15. Express each of the numerals below in base ten:

1. 10_{two} = ______ two(s) and ______ one(s) 1, 0

   Thus, $10_{two} = \_\_\_\__{ten}$ 2

2. 11_{two} = ______ two(s) and ______ one(s) 1, 1

   Thus, $11_{two} = \_\_\_\__{ten}$ 3

3. 101_{two} = ____ four(s), ____ two(s), and ____ one(s) 1, 0, 1

   Thus, $101_{two} = \_\_\_\__{ten}$ 5

4. 110_{two} = ____ four(s), ____ two(s), and ____ one(s) 1, 1, 0

   Thus, $110_{two} = \_\_\_\__{ten}$ 6

5. 111_{two} = ____ four(s), ____ two(s), and ____ one(s) 1, 1, 1

   Thus, $111_{two} = \_\_\_\__{ten}$ 7

16. The 1 in 1000_{two} is ________ times the 1 in 100_{two}. So 1000_{two} means 1 ——?——. two

a. four b. eight c. one thousand b. eight

17. Since $1000_{\text{two}} = 8_{\text{ten}}$, you can see that $1001_{\text{two}} =$ —?—.

a. 5_{ten} b. 9_{ten} c. 1001_{ten} — b. 9_{ten}

18. The numeral 1010_{two} tells you there are ______ eight(s), ______ four(s), ______ two(s), and ______ one(s). Thus, 1010_{two} is the same as ______ $_{\text{ten}}$.

1, 0
1, 0
10

19. $1011_{\text{two}} =$ ______ $_{\text{ten}}$. — 11

20. To transform a numeral from base ten into base two, or vice versa, we find out how many ones, twos, fours, eights, etc. the numeral represents.

For example, 9_{ten} represents ______ eight(s), ______ four(s), ______ two(s), and ______ one(s). Thus, $9_{\text{ten}} = 1001_{\text{two}}$.

1, 0
0, 1

21. In base ten, 12_{ten} has ______ eight(s), ______ four(s), ______ two(s) and ______ one(s). Thus, $12_{\text{ten}} =$ __________ $_{\text{two}}$.

1, 1, 0
0, 1100

22. The numeral 1101_{two} means ______ eight(s), ______ four(s), ______ two(s), and ______ one(s). And so, $1101_{\text{two}} =$ ______ $_{\text{ten}}$.

1, 1
0, 1, 13

23. In base two, the numeral 16_{ten} can be written as —?—.

a. 100_{two} b. 1000_{two} c. 10000_{two} — c. 10000_{two}

24. Translate each of the following from base two to base ten:

1. $1110_{\text{two}} =$ ______ $_{\text{ten}}$ — 14
2. $10001_{\text{two}} =$ ______ $_{\text{ten}}$ — 17
3. $11000_{\text{two}} =$ ______ $_{\text{ten}}$ — 24
4. $11011_{\text{two}} =$ ______ $_{\text{ten}}$ — 27
5. $100000_{\text{two}} =$ ______ $_{\text{ten}}$ — 32

25. Change each of the following numerals from base ten to base two:

1. $18_{ten} = \underline{\qquad\qquad}_{two}$ 10010

2. $21_{ten} = \underline{\qquad\qquad}_{two}$ 10101

3. $24_{ten} = \underline{\qquad\qquad}_{two}$ 11000

4. $31_{ten} = \underline{\qquad\qquad}_{two}$ 11111

5. $40_{ten} = \underline{\qquad\qquad}_{two}$ 101000

"Bulb Numerals"

26. One of the most important applications of base two is in the field of electronic computers. We can't go into any great detail about computer theory, but we can take a look at one simple application of base two in electronics.

You know that there are two positions for an electrical switch—off and on. You know also that we use only two digits in our base two system—0 and ______. 1

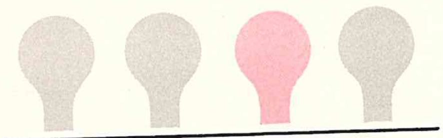
27. An electric bulb can be on, like this:

In this case, the bulb stands for the digit 1.

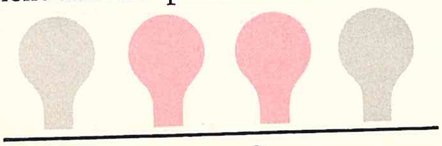
Or it can be off, like this:

In this case, the bulb stands for the digit ______. 0

28. Arrangements of bulbs can be used to represent numerals in base two.



The arrangement above represents the numeral 10_{two}.

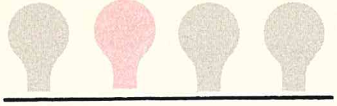


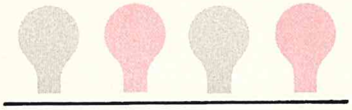
This arrangement represents ——?——.

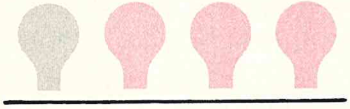
a. 110_{two} b. 100_{two} c. 101_{two}

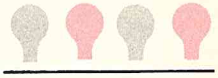
a. 110_{two}

29. To represent 101 $_{two}$, we would use ——?——.

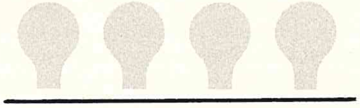
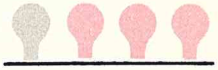
a. 

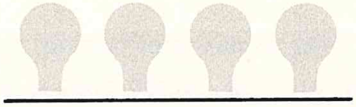
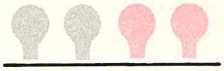
b. 

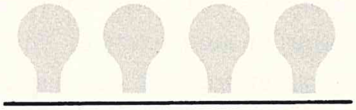
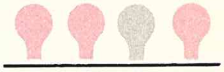
c. 

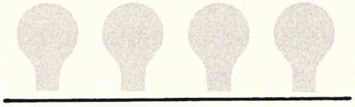
b. 

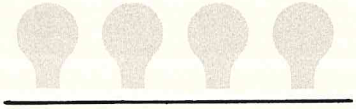
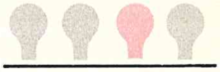
30. Shade or color the appropriate bulbs for each of the following:

1. 111 $_{two}$  

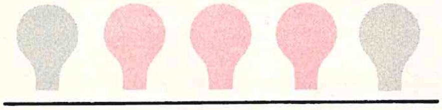
2. 11 $_{two}$  

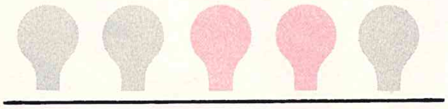
3. 1101 $_{two}$  

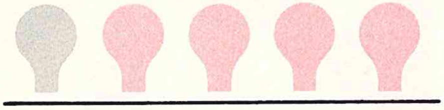
4. 1001 $_{two}$  

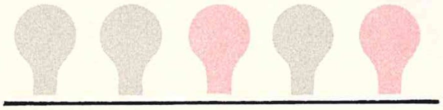
5. 10 $_{two}$  

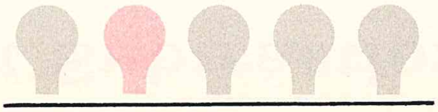
31. Translate each series of bulbs to a numeral in base two:

1.  ________ two 1110

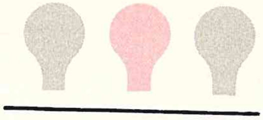
2.  ________ two 110

3.  ________ two 1111

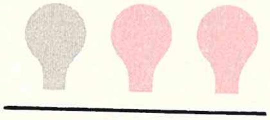
4.  ________ two 101

5.  ________ two 1000

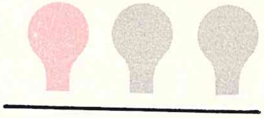
32. Now let's translate directly from "bulb numerals" to numerals in base ten. For example:



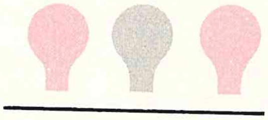
means 2 ten.



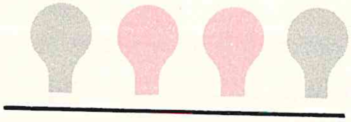
means 3 ten.



means 4 ten.

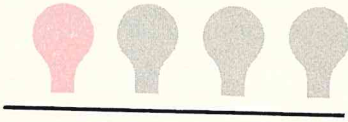
Then  means ____ ten. 5

33. The arrangement



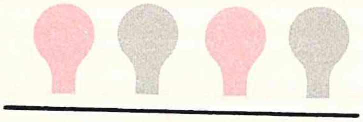
stands for ____ ten. 6

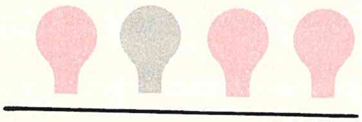
34. The arrangement

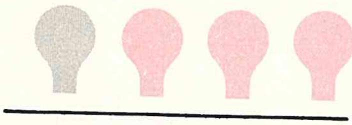


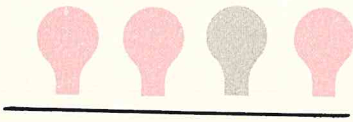
represents ____ ten. 8

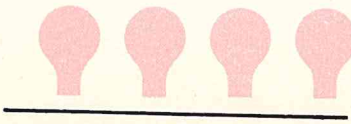
35. Translate each of the following "bulb numerals" into base ten:

1.  ____ ten 10

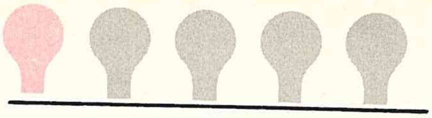
2.  ____ ten 11

3.  ____ ten 7

4.  ____ ten 13

5.  ____ ten 15

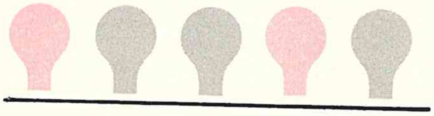
36. Translate this "bulb numeral" into a base ten numeral:



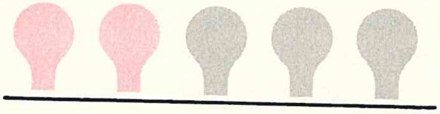
The bulbs represent ______ ten.

16

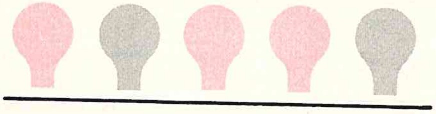
37. Now express in base ten each of the following "bulb numerals":

1.  ______ ten

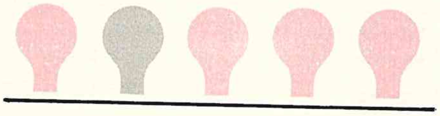
18

2.  ______ ten

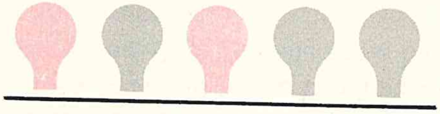
24

3.  ______ ten

22

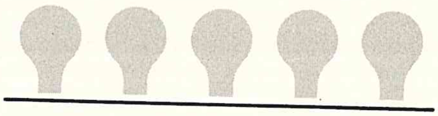
4.  ______ ten

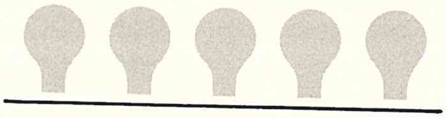
23

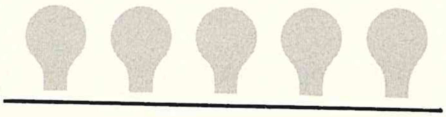
5.  ______ ten

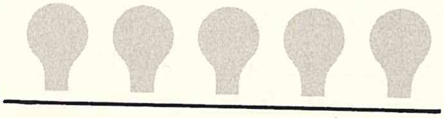
20

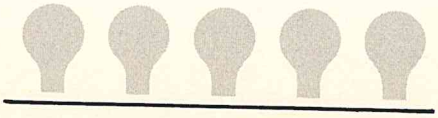
38. Now try translating base ten numerals into "bulb numerals." Shade the appropriate bulbs in each case:

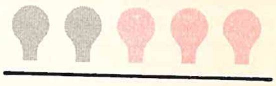
1. 7 ten 

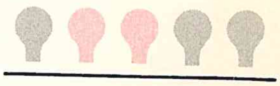
2. 12 ten 

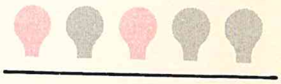
3. 20 ten 

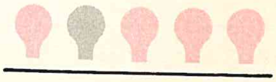
4. 23 ten 

5. 28 ten 









39. Now try these:

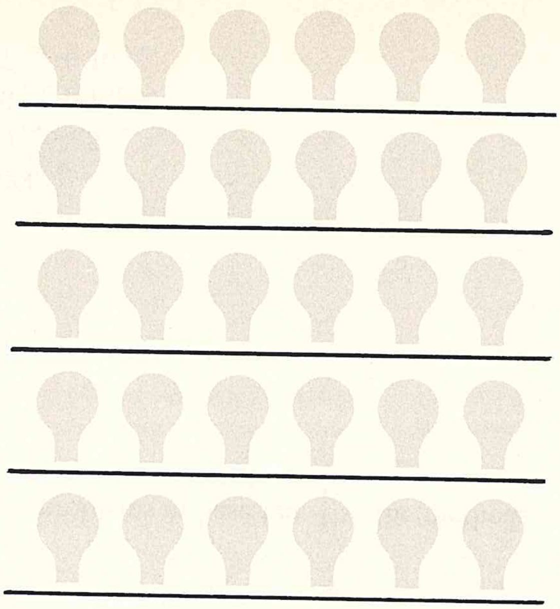
1. 29_{ten} __________

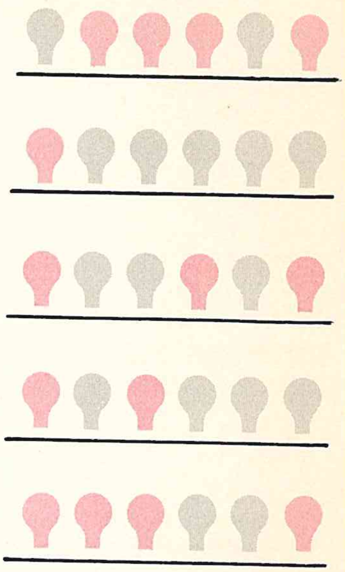
2. 32_{ten} __________

3. 37_{ten} __________

4. 40_{ten} __________

5. 57_{ten} __________





Addition in Base Two

40. Using "bulb numerals" is only one application of base two. We can also add, subtract, multiply, and divide in base two. Since there are only two digits in base two, 0 and 1, the four operations are quite easy.

Let's take a look at addition. In base two,
$1_{\text{two}} + 1_{\text{two}} =$ —?—.

a. 1_{two} **b.** 10_{two} **c.** 100_{two}

b. 10_{two}

41. $1_{\text{two}} + 1_{\text{two}} + 1_{\text{two}} =$ —?—.

a. 10_{two} **b.** 11_{two} **c.** 3_{two}

b. 11_{two}

42. $1_{\text{two}} + 1_{\text{two}} + 1_{\text{two}} + 1_{\text{two}} =$ ________ two.

100

43. Try each of these:

1. $10_{\text{two}} + 1_{\text{two}} =$ ________ two — 11
2. $11_{\text{two}} + 1_{\text{two}} =$ ________ two — 100
3. $100_{\text{two}} + 1_{\text{two}} =$ ________ two — 101
4. $100_{\text{two}} + 11_{\text{two}} =$ ________ two — 111
5. $101_{\text{two}} + 1_{\text{two}} =$ ________ two — 110
6. $101_{\text{two}} + 11_{\text{two}} =$ ________ two — 1000

44. In base two, $10_{two} + 10_{two} =$ ——?——.

a. 20_{two} **b.** 100_{two} **c.** 1000_{two} — **b.** 100_{two}

45. Then, $10_{two} + 11_{two} =$ ________ two. — 101

46. And $11_{two} + 11_{two} =$ ________ two. — 110

47. Here are some more addition problems with base two numerals:

1. $10_{two} + 11_{two} + 10_{two} =$ ________ two — 111
2. $11_{two} + 11_{two} + 11_{two} =$ ________ two — 1001
3. $10_{two} + 1_{two} + 1_{two} =$ ________ two — 100
4. $10_{two} + 10_{two} + 10_{two} =$ ________ two — 110
5. $11_{two} + 11_{two} + 11_{two} + 10_{two} =$ ________ two — 1011

48. Try this one in base two:

$$\begin{array}{r} 1010_{two} \\ +1110_{two} \\ \hline \_\_\_\_\_\_ \end{array}$$

11000

49.

$$\begin{array}{r} 11011_{two} \\ +1101_{two} \\ \hline \_\_\_\_\_\_ \end{array}$$

101000

50.

$$\begin{array}{r} 1101_{two} \\ 110_{two} \\ 1001_{two} \\ 111_{two} \\ +\quad 10_{two} \\ \hline \_\_\_\_\_\_ \end{array}$$

100101

51.

$$\begin{array}{r} 110010101_{two} \\ +10101101_{two} \\ \hline \_\_\_\_\_\__{two} \end{array}$$

1001000010

52. Now find each of the missing addends:

1. $10_{two} + \_\_\_\__{two} = 100_{two}$ 10
2. $11_{two} + \_\_\_\__{two} = 100_{two}$ 1
3. $100_{two} + \_\_\_\__{two} = 1000_{two}$ 100
4. $110_{two} + \_\_\_\__{two} = 1000_{two}$ 10
5. $1101_{two} + \_\_\_\__{two} = 10001_{two}$ 100
6. $\_\_\_\__{two} + 11_{two} = 100_{two}$ 1
7. $\_\_\_\__{two} + 11_{two} = 110_{two}$ 11
8. $\_\_\_\__{two} + 11_{two} = 111_{two}$ 100
9. $\_\_\_\__{two} + 101_{two} = 1000_{two}$ 11
10. $\_\_\_\__{two} + 111_{two} = 1001_{two}$ 10

Subtraction in Base Two

53. With certain combinations of digits, subtraction in base two is quite simple. For example, $1_{two} - 1_{two} = \_\_\_\__{two}$. And $1_{two} - 0_{two} = \_\_\_\__{two}$.

0

1

54. $10_{two} - 10_{two} = \_\_\_\__{two}$.

0

55. $10_{two} - 0_{two} = \_\_\_\__{two}$.

10

56. Now try these subtraction problems:

1. $11_{\text{two}} - 1_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 10
2. $11_{\text{two}} - 10_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 1
3. $101_{\text{two}} - 1_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 100
4. $110_{\text{two}} - 10_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 100
5. $111_{\text{two}} - 11_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 100

57. So far, the subtraction examples you have seen were quite easy. It was not difficult for you to find that $111_{\text{two}} - 11_{\text{two}} = 100_{\text{two}}$.

Now you are going to look at problems in which the equal additions method of subtraction can be very helpful. For example, you would probably want to use this method to find the answer to $100_{\text{two}} - 11_{\text{two}}$.

Another example of this type is ——?——.

a. $111_{\text{two}} - 10_{\text{two}}$ **b.** $1011_{\text{two}} - 10_{\text{two}}$ **c.** $101_{\text{two}} - 11_{\text{two}}$ — **c.** $101_{\text{two}} - 11_{\text{two}}$

58. Let's try to find the answer to $10_{\text{two}} - 1_{\text{two}}$. You know from Chapter 3 that we can add the same number to both 10_{two} and 1_{two} without changing the answer to the subtraction problem.

Suppose you add 1_{two} to both 10_{two} and 1_{two}. Then the subtraction problem becomes ——?——.

a. $11_{\text{two}} - 10_{\text{two}}$ **b.** $10_{\text{two}} - 10_{\text{two}}$ **c.** $11_{\text{two}} - 1_{\text{two}}$ — **a.** $11_{\text{two}} - 10_{\text{two}}$

59. The answer to this subtraction problem is $\_\_\_\_\_\__{\text{two}}$, and thus the 1

answer to the original problem, $10_{\text{two}} - 1_{\text{two}}$, is $\_\_\_\_\_\__{\text{two}}$. 1

60. Now let's find the answer to $100_{\text{two}} - 10_{\text{two}}$. If we use the method of equal additions, we can add 10_{two} to both 100_{two} and 10_{two} to get ——?——.

a. $100_{\text{two}} - 100_{\text{two}}$ **b.** $110_{\text{two}} - 100_{\text{two}}$ **c.** $110_{\text{two}} - 20_{\text{two}}$ — **b.** $110_{\text{two}} - 100_{\text{two}}$

61. The answer to this new problem is $\_\_\_\_\_\__{\text{two}}$, and thus the answer 10

to $100_{\text{two}} - 10_{\text{two}}$ is $\_\_\_\_\_\__{\text{two}}$. 10

62. Use the method of equal additions to find the answer to each of these subtraction problems:

1. $101_{two} - 11_{two} = \_\_\_\_\_\__{two}$ — 10
2. $110_{two} - 1_{two} = \_\_\_\_\_\__{two}$ — 101
3. $110_{two} - 11_{two} = \_\_\_\_\_\__{two}$ — 11
4. $1000_{two} - 111_{two} = \_\_\_\_\_\__{two}$ — 1

63. Sometimes you have to make two equal additions to arrive at the answer to a subtraction problem.

To find the answer to $100_{two} - 1_{two}$, we can add 1_{two} to both numbers. The resulting problem is ——?——.

a. $101_{two} - 2_{two}$ b. $101_{two} - 10_{two}$ c. $100_{two} - 10_{two}$ — b. $101_{two} - 10_{two}$

64. But the problem can be transformed again. If we add 10_{two} to both 101_{two} and 10_{two}, we get ——?——.

a. $111_{two} - 100_{two}$ b. $110_{two} - 100_{two}$ c. $111_{two} - 20_{two}$ — a. $111_{two} - 100_{two}$

65. You know that $111_{two} - 100_{two} = \_\_\_\_\_\__{two}$, and thus the answer — 11

to the original problem, $100_{two} - 1_{two}$, is $\_\_\_\_\_\__{two}$. — 11

66. Let's try this problem:

$$\begin{array}{r} 1000_{two} \\ -\ 101_{two} \\ \hline \end{array}$$

First we transform the problem to another problem:

$1000_{two} + \_\_\_\__{two} = 1001_{two}$ — 1

$101_{two} + \_\_\_\__{two} = 110_{two}$ — 1

Then we have:

$$\begin{array}{r} 1001_{two} \\ -110_{two} \\ \hline \end{array}$$

And we can transform this one:

$1001_{two} + \_\_\_\__{two} = 1011_{two}$ — 10

$110_{two} + \_\_\_\__{two} = 1000_{two}$ — 10

And we have:

$$\begin{array}{r} 1011_{two} \\ -1000_{two} \\ \hline \end{array}$$

Since $1011_{two} - 1000_{two} = \_\_\_\_\_\__{two}$, we know that — 11

$1000_{two} - 101_{two} = \_\_\_\_\_\__{two}$. — 11

67. Instead of adding 1_{two} and then 10_{two} in the last problem, we could have combined two steps into one by adding —?— right away.

a. 11_{two} **b.** 100_{two} **c.** 1000_{two} — **a.** 11_{two}

68. Try this one:

1000_{two}	1010_{two}	$\_\_\_\_\_\__{two}$	1110
-10_{two}	$-\_\_\_\_\_\__{two}$	-1000_{two}	100

Solving this last problem, you can see that

$1000_{two} - 10_{two} = \_\_\_\_\_\__{two}$. 110

69. Here's another one:

1010_{two}	1011_{two}	$\_\_\_\_\_\__{two}$	1101
-101_{two}	$-\_\_\_\_\_\__{two}$	-1000_{two}	110

Since $1101_{two} - 1000_{two} = \_\_\_\_\_\__{two}$, you know that 101

$1010_{two} - 101_{two} = \_\_\_\_\_\__{two}$. 101

70. Try this subtraction problem:

$$\begin{array}{r} 100_{two} \\ -10_{two} \\ \hline \end{array}$$

$\_\_\_\__{two}$ 10

71. Now try this one:

$$\begin{array}{r} 1101_{two} \\ -110_{two} \\ \hline \end{array}$$

$\_\_\_\__{two}$ 111

72. Here's another one:

$$\begin{array}{r} 10101_{two} \\ -1101_{two} \\ \hline \end{array}$$

$\_\_\_\__{two}$ 1000

73. Try each of the following subtraction problems:

$$\begin{array}{r} 1000_{\text{two}} \\ -110_{\text{two}} \\ \hline \end{array}$$

_____ two 10

$$\begin{array}{r} 1001_{\text{two}} \\ -110_{\text{two}} \\ \hline \end{array}$$

_____ two 11

$$\begin{array}{r} 10001_{\text{two}} \\ -1010_{\text{two}} \\ \hline \end{array}$$

_____ two 111

$$\begin{array}{r} 10000_{\text{two}} \\ -1111_{\text{two}} \\ \hline \end{array}$$

_____ two 1

$$\begin{array}{r} 1110_{\text{two}} \\ -111_{\text{two}} \\ \hline \end{array}$$

_____ two 111

Multiplication in Base Two

74. Now let's take a look at multiplication in base two. The list of basic multiplication facts is very simple.

1. $0_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
2. $1_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
3. $0_{\text{two}} \times 1_{\text{two}} =$ _____ two 0
4. $1_{\text{two}} \times 1_{\text{two}} =$ _____ two 1

75. If we multiply any number by zero, the answer is zero.

1. $10_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
2. $11_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
3. $100_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
4. $101_{\text{two}} \times 0_{\text{two}} =$ _____ two 0
5. $111_{\text{two}} \times 0_{\text{two}} =$ _____ two 0

76. If we multiply any number by one, the answer is the number itself.

1. $10_{two} \times 1_{two} =$ ________ $_{two}$ — 10
2. $11_{two} \times 1_{two} =$ ________ $_{two}$ — 11
3. $100_{two} \times 1_{two} =$ ________ $_{two}$ — 100
4. $101_{two} \times 1_{two} =$ ________ $_{two}$ — 101
5. $111_{two} \times 1_{two} =$ ________ $_{two}$ — 111

77. Multiplication by 10_{two} is quite easy. For example,
$11_{two} \times 10_{two} =$ —?—.

a. 11_{two} b. 110_{two} c. 1100_{two} — b. 110_{two}

78. And $111_{two} \times 10_{two} =$ ________ $_{two}$. — 1110

79. $10_{two} \times 10_{two} =$ —?—.

a. 10_{two} b. 100_{two} c. 1000_{two} — b. 100_{two}

80. $100_{two} \times 10_{two} =$ ________ $_{two}$. — 1000

81. $1010_{two} \times 10_{two} =$ ________ $_{two}$. — 10100

82. Multiplication by 100_{two} is just as easy. In base two,
$11_{two} \times 100_{two} =$ —?—.

a. 110_{two} b. 1100_{two} c. 11000_{two} — b. 1100_{two}

83. $1010_{two} \times 100_{two} =$ ________ $_{two}$. — 101000_{two}

84. Try the following multiplication problems in base two:

1. $101_{two} \times 100_{two} = \_\_\_\_\_\_\__{two}$ 10100

2. $110_{two} \times 10_{two} = \_\_\_\_\_\_\__{two}$ 1100

3. $100_{two} \times 100_{two} = \_\_\_\_\_\_\__{two}$ 10000

4. $111_{two} \times 100_{two} = \_\_\_\_\_\_\__{two}$ 11100

5. $100_{two} \times 10_{two} = \_\_\_\_\_\_\__{two}$ 1000

85. Now try these multiplication problems in base two:

1. $101_{two} \times 1000_{two} = \_\_\_\_\_\_\__{two}$ 101000

2. $111_{two} \times 1000_{two} = \_\_\_\_\_\_\__{two}$ 111000

3. $11_{two} \times 100000_{two} = \_\_\_\_\_\_\__{two}$ 1100000

4. $11010_{two} \times 1000_{two} = \_\_\_\_\_\_\__{two}$ 11010000

5. $11100011_{two} \times 1000_{two} = \_\_\_\_\_\_\__{two}$ 11100011000

86. To multiply 11_{two} by 11_{two}, we can express one of the 11_{two}'s as

$10_{two} + \_\_\_\__{two}$. 1

Then we have two partial products:

$$11_{two} \times 11_{two} = \begin{cases} 1_{two} \times 11_{two} = \_\_\_\_\_\__{two} \\ 10_{two} \times 11_{two} = \_\_\_\_\_\__{two} \end{cases}$$ 11 110

The sum of the partial products is $\_\_\_\_\_\__{two}$, and thus 1001

$11_{two} \times 11_{two} = \_\_\_\_\_\__{two}$. 1001

87. We could write $11_{two} \times 11_{two}$ in this form:

$$\begin{array}{rr} & 11_{two} \\ & \times 11_{two} \\ \hline 1_{two} \times \_\_\_\__{two} = & 11_{two} \\ 10_{two} \times \_\_\_\__{two} = & 110_{two} \\ \hline & 1001_{two} \end{array}$$ 11 11

88. Here's another problem:

$$\begin{array}{r} 101_{\text{two}} \\ \times 11_{\text{two}} \\ \hline \end{array}$$

$1_{\text{two}} \times 101_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 101

$10_{\text{two}} \times 101_{\text{two}} = 1010_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 1111

89.

$$\begin{array}{r} 111_{\text{two}} \\ \times 11_{\text{two}} \\ \hline \end{array}$$

$1_{\text{two}} \times 111_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 111

$10_{\text{two}} \times 111_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 1110

$\_\_\_\_\_\__{\text{two}}$ 10101

90.

$$\begin{array}{r} 111_{\text{two}} \\ \times 101_{\text{two}} \\ \hline \end{array}$$

$1_{\text{two}} \times 111_{\text{two}} = 111_{\text{two}}$

$100_{\text{two}} \times 111_{\text{two}} = 11100_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 100011

91.

$$\begin{array}{r} 111_{\text{two}} \\ \times 110_{\text{two}} \\ \hline \end{array}$$

$10_{\text{two}} \times 111_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 1110

$100_{\text{two}} \times 111_{\text{two}} = \_\_\_\_\_\__{\text{two}}$ 11100

$\_\_\_\_\_\__{\text{two}}$ 101010

92.

$$\begin{array}{r} 1001_{\text{two}} \\ \times 11_{\text{two}} \\ \hline \end{array}$$

$\_\_\_\_\_\__{\text{two}}$ 1001

$\_\_\_\_\_\__{\text{two}}$ 10010

$\_\_\_\_\_\__{\text{two}}$ 11011

93. 1101_{two}
$\times 101_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 1101

$\_\_\_\_\_\__{\text{two}}$ 110100

$\_\_\_\_\_\__{\text{two}}$ 1000001

94. 110_{two}
$\times 101_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 11110

95. 101_{two}
$\times 101_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 11001

96. 111_{two}
$\times 111_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 110001

97. 1111_{two}
$\times 110_{\text{two}}$

$\_\_\_\_\_\__{\text{two}}$ 1011010

Division in Base Two

98. Now let's turn to division in base two. You will find that it is very easy to divide using base two numerals.
For example:

$10_{two}\overline{)\,1000_{two}}$

$\_\_\_\_\_\__{two}$ 100_{two} 1000

0_{two}

Then $1000_{two} \div 10_{two} = \_\_\_\_\_\__{two}$. 100

99. Let's try this problem:

$10_{two}\overline{)\,1010_{two}}$

$\_\_\_\_\_\__{two}$ 100_{two} 1000

10_{two}

10_{two} $\_\_\_\__{two}$ 1

0_{two}

The sum of the trial quotients is ______, and thus 101

$1010_{two} \div 10_{two} = \_\_\_\_\_\__{two}$. 101

100. Here's another one:

$11_{two}\overline{)\,1111_{two}}$

1100_{two} $\_\_\_\__{two}$ 100

$\_\_\_\__{two}$ 11

11_{two} 1_{two}

0_{two}

Then $1111_{two} \div 11_{two} = \_\_\_\_\_\__{two}$. 101

101. $11_{two}\overline{)\,11110_{two}}$

$\_\_\_\_\_\__{two}$ 1000_{two} 11000

110_{two}

$\_\_\_\__{two}$ $\_\_\_\__{two}$ 110, 10

0_{two}

Then $11110_{two} \div 11_{two} = \_\_\_\_\_\__{two}$. 1010

102. $11_{two} \overline{)1001_{two}}$

$\underline{\underline{\quad\quad}}_{two}$ 10_{two} 110

11_{two}

$\underline{11_{two}}$ $\_\_\__{two}$ 1

0_{two}

Then $1001_{two} \div 11_{two} = \_\_\_\__{two}$. 11

103. $101_{two} \overline{)11001_{two}}$

$\underline{\underline{\quad\quad}}_{two}$ 100_{two} 10100

$\_\_\_\__{two}$ 101

$\underline{\quad\quad}_{two}$ 1_{two} 101

0_{two}

Then $11001_{two} \div 101_{two} = \_\_\_\__{two}$. 101

04. $110_{two} \overline{)10010_{two}}$

$\underline{\underline{\quad\quad}}_{two}$ $\_\_\__{two}$ 1100, 10

$\_\_\_\__{two}$ 110

$\underline{\quad\quad}_{two}$ $\_\_\__{two}$ 110, 1

$\_\_\_\__{two}$ 0

Then $10010_{two} \div 110_{two} = \_\_\_\__{two}$. 11

05. $111_{two} \overline{)1000110_{two}}$

$\underline{\underline{\quad\quad}}_{two}$ $\_\_\__{two}$ 111000, 1000

$\_\_\_\__{two}$ 1110

$\underline{\quad\quad}_{two}$ $\_\_\__{two}$ 1110, 10

$\_\_\_\__{two}$ 0

Then $1000110_{two} \div 111_{two} = \_\_\_\__{two}$. 1010

106. $101_{two}\overline{)\,110010_{two}}$

$110010_{two} \div 101_{two} =$ __________ two. 1010

107. $111_{two}\overline{)\,101010_{two}}$

$101010_{two} \div 111_{two} =$ ________ two. 110

108. $1001_{two}\overline{)\,11011_{two}}$

$11011_{two} \div 1001_{two} =$ ______ two. 11

109. $11_{two}\overline{)\,111001_{two}}$

$111001_{two} \div 11_{two} =$ __________ two. 10011

110. $1101_{two}\overline{)\,1110101_{two}}$

$1110101_{two} \div 1101_{two} =$ __________ two. 1001

Try Your Hand

■ You have probably found that base two is one of the simplest bases of all. Try these problems to sharpen your skill in base two. The answers appear on page 171.

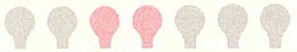
1. In base ten, tell how many items your mother wanted if she sent you to the store for:

   a. 11_{two} pounds of sugar
   b. 101_{two} ears of corn
   c. 111_{two} heads of cabbage
   d. 1000_{two} slices of salami
   e. 1010_{two} pounds of raisins
   f. 1011_{two} hard rolls
   g. 1111_{two} tomatoes
   h. 10000_{two} quarts of milk
   i. 10010_{two} boxes of cereal
   j. 11111_{two} teabags

2. Translate these base ten numerals to base two:

   a. 4_{ten}
   b. 7_{ten}
   c. 12_{ten}
   d. 14_{ten}
   e. 18_{ten}
   f. 25_{ten}
   g. 32_{ten}
   h. 35_{ten}
   i. 65_{ten}
   j. 70_{ten}

3. Translate these base two "bulb numerals" into base ten.

   a.  ________
   b.  ________
   c.  ________
   d.  ________
   e.  ________
   f.  ________
   g.  ________
   h.  ________
   i.  ________
   j.  ________

4. In base two, count:

   a. by twos from 10_{two} to 110_{two}
   b. by threes from 1_{two} to 111_{two}
   c. by fours from 10_{two} to 1110_{two}
   d. backwards by twos from 1000_{two} to 10_{two}
   e. backwards by fours from 1011_{two} to 11_{two}

5. Solve the following base two addition problems:

a. $101_{\text{two}} + 10_{\text{two}}$

b. $101_{\text{two}} + 11_{\text{two}}$

c. $111_{\text{two}} + 11_{\text{two}}$

d. $1011_{\text{two}} + 110_{\text{two}}$

e. $1111_{\text{two}} + 101_{\text{two}}$

6. Find answers to these subtraction problems:

a. $101_{\text{two}} - 11_{\text{two}}$

b. $110_{\text{two}} - 11_{\text{two}}$

c. $1000_{\text{two}} - 111_{\text{two}}$

d. $1001_{\text{two}} - 110_{\text{two}}$

e. $10000_{\text{two}} - 101_{\text{two}}$

7. Try each of these multiplication problems in base two:

a. $11_{\text{two}} \times 11_{\text{two}}$

b. $101_{\text{two}} \times 11_{\text{two}}$

c. $101_{\text{two}} \times 101_{\text{two}}$

d. $111_{\text{two}} \times 110_{\text{two}}$

e. $111_{\text{two}} \times 111_{\text{two}}$

8. Find the answers to these division problems in base two:

a. $1001_{\text{two}} \div 11_{\text{two}}$

b. $11001_{\text{two}} \div 101_{\text{two}}$

c. $11000_{\text{two}} \div 100_{\text{two}}$

d. $100011_{\text{two}} \div 111_{\text{two}}$

e. $10101_{\text{two}} \div 111_{\text{two}}$

THINK IT OVER

Try answering these questions on base two. You'll find that some of the questions ask you to extend and develop the ideas covered in this chapter.

1. If you had to choose between base five and base two, which would you select? Why?

2. Why do you think base two is important in industry? Why do many computers use base two?

3. How can you tell when a number in base two is divisible by two? How can you tell when a number in base two is divisible by four? How can you tell when a number in base two is divisible by two but not by four?

7 A Look at Base Twelve

A REPLACEMENT FOR BASE TEN?

None of the bases you have seen so far in this program has been greater than ten. You have worked in base two, three, four, and so forth up to ten. Now you are going to learn about a base which is greater than ten. You are going to learn about **base twelve.**

In some respects, base twelve is very different from the other bases you have seen. In other respects, it is quite the same. As you work through this chapter, try to determine whether or not you prefer base twelve to base ten. For some years now, the Duodecimal Society of America has spearheaded a movement to replace base ten with base twelve.

Using Base Twelve Numerals

1. In base two, you use two digits to represent numbers. In base five,

you use __________ digits to represent numbers. five

Now, if you use base twelve, you would need ____________ digits. twelve

2. We already have ten digits from base ten that we can use in base

twelve. These digits are: 0, 1, 2, 3, 4, 5, 6, ____, ____, and ____. 7, 8, 9

3. Then, to represent numbers in base twelve, we need two more digits. We need a digit for ten, and another for eleven. Let's agree to use **T** for ten and **E** for eleven.

In base twelve, then, $9_{\text{twelve}} + 1_{\text{twelve}} = T_{\text{twelve}}$ and

$T_{\text{twelve}} + 1_{\text{twelve}} =$ ______ $_{\text{twelve}}$. E

4. Transform the following base twelve numerals into base ten:

1. $0_{\text{twelve}} =$ ______ $_{\text{ten}}$ 0
2. $1_{\text{twelve}} =$ ______ $_{\text{ten}}$ 1
3. $2_{\text{twelve}} =$ ______ $_{\text{ten}}$ 2
4. $3_{\text{twelve}} =$ ______ $_{\text{ten}}$ 3
5. $4_{\text{twelve}} =$ ______ $_{\text{ten}}$ 4
6. $5_{\text{twelve}} =$ ______ $_{\text{ten}}$ 5
7. $6_{\text{twelve}} =$ ______ $_{\text{ten}}$ 6
8. $7_{\text{twelve}} =$ ______ $_{\text{ten}}$ 7
9. $8_{\text{twelve}} =$ ______ $_{\text{ten}}$ 8
10. $9_{\text{twelve}} =$ ______ $_{\text{ten}}$ 9
11. $T_{\text{twelve}} =$ ______ $_{\text{ten}}$ 10
12. $E_{\text{twelve}} =$ ______ $_{\text{ten}}$ 11

5. In base twelve, 10_{twelve} means ______ twelve(s) and ______ 1, 0

one(s). That is, $10_{\text{twelve}} =$ ______ $_{\text{ten}}$. 12

6. The numeral 15_{twelve} means ______ twelve(s) and ______ one(s). 1, 5

Thus $15_{\text{twelve}} =$ ______ $_{\text{ten}}$. 17

7. In base twelve, 19_{twelve} means ______ twelve(s) and ______ 1, 9

one(s). That is, $19_{\text{twelve}} =$ ______ $_{\text{ten}}$. 21

8. Of course, $1T_{\text{twelve}}$ means ______ twelve(s) and ______ one(s). 1, 10

So $1T_{\text{twelve}} =$ ______ $_{\text{ten}}$. 22

9. The numeral in base twelve that represents 24_{ten} is —?—.

a. 24_{twelve} **b.** 20_{twelve} **c.** $2E_{\text{twelve}}$ **b.** 20_{twelve}

10. $T7_{\text{twelve}}$ means ______ twelve(s) and ______ one(s). Thus 10, 7

$T7_{\text{twelve}} =$ ______ $_{\text{ten}}$. 127

11. Transform the following numerals from base twelve to base ten:

1. $52_{\text{twelve}} =$ ______ $_{\text{ten}}$ 62
2. $E5_{\text{twelve}} =$ ______ $_{\text{ten}}$ 137
3. $6T_{\text{twelve}} =$ ______ $_{\text{ten}}$ 82
4. $TT_{\text{twelve}} =$ ______ $_{\text{ten}}$ 130
5. $E9_{\text{twelve}} =$ ______ $_{\text{ten}}$ 141

12. In base five, the 1 in 10_{five} is —?— times the 1 in 1_{five}.

a. one **b.** five **c.** twelve **b.** five

13. And the 1 in 100_{five} is —?— times the 1 in 10_{five}.

a. one **b.** five **c.** twelve **b.** five

14. In base twelve, the 1 in 10_{twelve} is —?— times the 1 in 1_{twelve}.

a. one **b.** five **c.** twelve **c.** twelve

15. And the 1 in 100_{twelve} is —?— times the 1 in 10_{twelve}.

a. one **b.** five **c.** twelve **c.** twelve

16. You know that 10 $_{\text{twelve}}$ means 1 twelve. So you can see that 100 $_{\text{twelve}}$ means 1 ——?——.

a. twelve b. one hundred c. one hundred forty-four

c. one hundred forty-four

17. The 2 in the numeral 12E $_{\text{twelve}}$ represents ——?——.

a. 2 ones b. 2 twelves c. 2 one hundred forty-fours

b. 2 twelves

18. The 5 in the numeral 5E6 $_{\text{twelve}}$ represents ——?——.

a. 5 ones b. 5 twelves c. 5 one hundred forty-fours

c. 5 one hundred forty-fours

19. In base twelve, 124 $_{\text{twelve}}$ means ______ one hundred forty-four(s), ______ twelve(s), and ______ one(s). Thus, 124 $_{\text{twelve}}$ = ______ $_{\text{ten}}$.

1
2, 4, 172

20. And E02 $_{\text{twelve}}$ means ______ one hundred forty-four(s), ______ twelve(s), and ______ one(s). Thus, E02 $_{\text{twelve}}$ = ______ $_{\text{ten}}$.

11, 0
2, 1586

21. Transform the following base twelve numerals into base ten:

1. 200 $_{\text{twelve}}$ = ______ $_{\text{ten}}$ — 288
2. T0T $_{\text{twelve}}$ = ______ $_{\text{ten}}$ — 1450
3. 11E $_{\text{twelve}}$ = ______ $_{\text{ten}}$ — 167
4. 2TT $_{\text{twelve}}$ = ______ $_{\text{ten}}$ — 418
5. T0E $_{\text{twelve}}$ = ______ $_{\text{ten}}$ — 1451

22. Changing numerals from base ten to base twelve is not very difficult. To change 37 $_{\text{ten}}$ to base twelve, we must see that in 37 $_{\text{ten}}$ there are ______ twelve(s) and ______ one(s). Then 37 $_{\text{ten}}$ = ______ $_{\text{twelve}}$.

3, 1
31

23. In 68 $_{ten}$ there are ______ twelve(s) and ______ one(s). Thus, 5, 8

68 $_{ten}$ = ______ $_{twelve}$. 58

24. You know that in 58 $_{ten}$ there are ______ twelve(s) and ______ 4, 10

one(s). Thus, 58 $_{ten}$ = ______ $_{twelve}$. 4T

25. Now transform the following base ten numerals into base twelve:

1. 14 $_{ten}$ = ______ $_{twelve}$ 12
2. 22 $_{ten}$ = ______ $_{twelve}$ 1T
3. 40 $_{ten}$ = ______ $_{twelve}$ 34
4. 59 $_{ten}$ = ______ $_{twelve}$ 4E
5. 125 $_{ten}$ = ______ $_{twelve}$ T5

26. When the base ten number is larger than 143 $_{ten}$, we must find out how many one hundred forty-fours, twelves, and ones it contains.

For example, in 145 $_{ten}$ there are ______ one hundred forty- 1

four(s), ______ twelve(s), and ______ one(s). Thus, 0, 1

145 $_{ten}$ = ______ $_{twelve}$. 101

27. In 157 $_{ten}$ there are ______ one hundred forty-four(s), ______ 1, 1

twelve(s), and ______ one(s). Thus, 157 $_{ten}$ = ______ $_{twelve}$. 1, 111

28. Change each of the following from base ten to base twelve:

1. 150 $_{ten}$ = ______ $_{twelve}$ 106
2. 179 $_{ten}$ = ______ $_{twelve}$ 12E
3. 200 $_{ten}$ = ______ $_{twelve}$ 148
4. 360 $_{ten}$ = ______ $_{twelve}$ 260
5. 1439 $_{ten}$ = ______ $_{twelve}$ 9EE

Addition in Base Twelve

29. Addition in base twelve is quite easy to learn. Try these:

1. $2_{\text{twelve}} + 2_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 4
2. $3_{\text{twelve}} + 5_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 8
3. $5_{\text{twelve}} + 4_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 9
4. $2_{\text{twelve}} + 6_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 8
5. $4_{\text{twelve}} + 1_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 5

30. Of course, $5_{\text{twelve}} + 5_{\text{twelve}} = T_{\text{twelve}}$. Then

$5_{\text{twelve}} + 6_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$. — E

31. Find each of the following sums in base twelve:

1. $5_{\text{twelve}} + 4_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 9
2. $8_{\text{twelve}} + 3_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — E
3. $1_{\text{twelve}} + T_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — E
4. $3_{\text{twelve}} + 5_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 8
5. $6_{\text{twelve}} + 4_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — T

32. The sum of 6_{twelve} and 6_{twelve} is —?—.

a. 12_{twelve} **b.** 10_{twelve} **c.** T_{twelve} — **b.** 10_{twelve}

33. Find the sum of each of the following:

1. $5_{\text{twelve}} + 7_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 10
2. $T_{\text{twelve}} + 2_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 10
3. $3_{\text{twelve}} + 9_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 10
4. $4_{\text{twelve}} + 8_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 10
5. $E_{\text{twelve}} + 1_{\text{twelve}} = \underline{\quad}_{\text{twelve}}$ — 10

34. Since $8_{\text{twelve}} + 4_{\text{twelve}} = 10_{\text{twelve}}$, $8_{\text{twelve}} + 6_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 12

35. Since $E_{\text{twelve}} + 1_{\text{twelve}} = 10_{\text{twelve}}$; $E_{\text{twelve}} + 7_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 16

36. In base twelve, $T_{\text{twelve}} + 5_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 13

37. Find the sum of each of the following:

1. $9_{\text{twelve}} + 5_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 12
2. $8_{\text{twelve}} + E_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 17
3. $8_{\text{twelve}} + 8_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 14
4. $T_{\text{twelve}} + 9_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 17
5. $E_{\text{twelve}} + T_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 19

38. Since you know that $5_{\text{twelve}} + 3_{\text{twelve}} = 8_{\text{twelve}}$, you can see that

$35_{\text{twelve}} + 3_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 38

39. Since $9_{\text{twelve}} + 9_{\text{twelve}} = 16_{\text{twelve}}$, $49_{\text{twelve}} + 9_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 56

40. Since $T_{\text{twelve}} + 7_{\text{twelve}} = 15_{\text{twelve}}$, $5T_{\text{twelve}} + 7_{\text{twelve}} =$ ______ $_{\text{twelve}}$. 65

41. In base twelve:

1. $38_{\text{twelve}} + 6_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 42
2. $49_{\text{twelve}} + T_{\text{twelve}} =$ ______ $_{\text{twelve}}$ 57

42. Again in base twelve:

1. $27_{\text{twelve}} + E_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 36

2. $86_{\text{twelve}} + 9_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 93

43. Find each of the following sums:

1. $73_{\text{twelve}} + T_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 81

2. $14_{\text{twelve}} + 9_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 21

3. $3T_{\text{twelve}} + 8_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 46

4. $66_{\text{twelve}} + 6_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 70

5. $7T_{\text{twelve}} + E_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 89

44. The sum of 93_{twelve} and 8_{twelve} is ——?——.

a. $9E_{\text{twelve}}$ **b.** 101_{twelve} **c.** $T1_{\text{twelve}}$ **a.** $9E_{\text{twelve}}$

45. $9E_{\text{twelve}} + 1_{\text{twelve}} =$ ——?——.

a. 10_{twelve} **b.** 120_{twelve} **c.** $T0_{\text{twelve}}$ **c.** $T0_{\text{twelve}}$

46. $96_{\text{twelve}} + 8_{\text{twelve}}$ ——?——.

a. 104_{twelve} **b.** $T4_{\text{twelve}}$ **c.** $T2_{\text{twelve}}$ **c.** $T2_{\text{twelve}}$

47. Find the sums below:

1. $95_{\text{twelve}} + 7_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ T0

2. $98_{\text{twelve}} + 8_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ T4

3. $T3_{\text{twelve}} + 8_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ TE

4. $TT_{\text{twelve}} + 2_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ E0

5. $T7_{\text{twelve}} + T_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ E5

48. Now try these:

1. $347_{\text{twelve}} + T_{\text{twelve}} =$ ______ twelve — 355
2. $804_{\text{twelve}} + 9_{\text{twelve}} =$ ______ twelve — 811
3. $ET7_{\text{twelve}} + 6_{\text{twelve}} =$ ______ twelve — EE1
4. $766_{\text{twelve}} + E_{\text{twelve}} =$ ______ twelve — 775
5. $43T_{\text{twelve}} + E_{\text{twelve}} =$ ______ twelve — 449

49. In base twelve, $EE_{\text{twelve}} + 1_{\text{twelve}} = 100_{\text{twelve}}$. Then the sum of EE_{twelve} and 5_{twelve} is ______ twelve. — 104

50. $EE_{\text{twelve}} + 1_{\text{twelve}} = 100_{\text{twelve}}$.

Then $6EE_{\text{twelve}} + 8_{\text{twelve}} =$ ______ twelve. — 707

51. And $9EE_{\text{twelve}} + 1_{\text{twelve}} = T00_{\text{twelve}}$.

Thus $9EE_{\text{twelve}} + T_{\text{twelve}} =$ ______ twelve. — T09

52. Try these addition problems:

1. $EE_{\text{twelve}} + 7_{\text{twelve}} =$ ______ twelve — 106
2. $5E8_{\text{twelve}} + T_{\text{twelve}} =$ ______ twelve — 606
3. $ET_{\text{twelve}} + 7_{\text{twelve}} =$ ______ twelve — 105
4. $9E9_{\text{twelve}} + 4_{\text{twelve}} =$ ______ twelve — T01
5. $5E8_{\text{twelve}} + 8_{\text{twelve}} =$ ______ twelve — 604

53. In base twelve, $4_{\text{twelve}} + 5_{\text{twelve}} = 9_{\text{twelve}}$.

Thus $40_{\text{twelve}} + 50_{\text{twelve}} =$ ______ twelve. — 90

54. Since $8_{\text{twelve}} + 7_{\text{twelve}} = 13_{\text{twelve}}$,

$80_{\text{twelve}} + 70_{\text{twelve}} =$ ______ twelve. — 130

55. And since $T_{twelve} + 7_{twelve} = 15_{twelve}$,

$T0_{twelve} + 70_{twelve} =$ ________ twelve. 150

56. Find the sum of each of the following:

1. $10_{twelve} + 90_{twelve} =$ ______ twelve T0
2. $60_{twelve} + 50_{twelve} =$ ______ twelve E0
3. $70_{twelve} + T0_{twelve} =$ ______ twelve 150
4. $T00_{twelve} + 100_{twelve} =$ ______ twelve E00
5. $800_{twelve} + 500_{twelve} =$ ______ twelve 1100

57. $21_{twelve} + 43_{twelve}$ means 2 twelves and 1 one plus 4 twelves and 3 ones. Altogether, there are ______ twelve(s) and ______ one(s). Thus $21_{twelve} + 43_{twelve} =$ ______ twelve. 6, 4 64

58. $39_{twelve} + 18_{twelve}$ means 3 twelves and 9 ones plus 1 twelve and 8 ones. Altogether, there are 4 twelves and 17 ones, but in base twelve we would express this as ______ twelve(s) and ______ one(s). Thus $39_{twelve} + 18_{twelve} =$ ______ twelve. 5, 5 55

59. In base twelve:

$$\begin{array}{r} 35_{twelve} \\ +26_{twelve} \\ \hline \text{\_\_\_\_\_}_{twelve} \end{array}$$

5E

60.

$$\begin{array}{r} 65_{twelve} \\ +46_{twelve} \\ \hline \text{\_\_\_\_\_}_{twelve} \end{array}$$

TE

61.

$$\begin{array}{r} 154_{twelve} \\ +29_{twelve} \\ \hline \text{\_\_\_\_\_}_{twelve} \end{array}$$

181

62. $\begin{array}{r} 307_{\text{twelve}} \\ +196_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 4T1

63. $\begin{array}{r} 58_{\text{twelve}} \\ +73_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 10E

64. $\begin{array}{r} 294_{\text{twelve}} \\ +135_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 409

65. $\begin{array}{r} 475_{\text{twelve}} \\ +258_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 711

66. $\begin{array}{r} 10T4_{\text{twelve}} \\ +2238_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 3320

Subtraction in Base Twelve

67. Subtraction in base twelve is not much different from subtraction in other bases, except that you have to be careful using the digits T and E.

In base twelve:

$\begin{array}{r} 3TT_{\text{twelve}} \\ -62_{\text{twelve}} \\ \hline ? \end{array}$

a. 348_{twelve} b. $3E9_{\text{twelve}}$ c. 248_{twelve} a. 348_{twelve}

68. $\begin{array}{r} ET_{\text{twelve}} \\ -77_{\text{twelve}} \\ \hline \end{array}$

$\_\_\_\__{\text{twelve}}$ 43

69.

$$\begin{array}{r} \text{TET}_{\text{twelve}} \\ -899_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$$

221

70.

$$\begin{array}{r} \text{3ET}_{\text{twelve}} \\ -\text{2T}_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$$

390

71.

$$\begin{array}{r} \text{1EEE}_{\text{twelve}} \\ -\text{ET9}_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$$

1012

72.

$$\begin{array}{r} \text{T43E8}_{\text{twelve}} \\ -\text{510T3}_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$$

53315

73. In subtracting E $_{\text{twelve}}$ from 25 $_{\text{twelve}}$, you can use the equal additions method of subtraction. If we add 1 $_{\text{twelve}}$ to both 25 $_{\text{twelve}}$ and E $_{\text{twelve}}$, we have:

$$\begin{array}{r} 25_{\text{twelve}} \\ -\text{E}_{\text{twelve}} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 26_{\text{twelve}} \\ -\_\_\_\__{\text{twelve}} \\ \hline \end{array}$$

10

The answer to this simpler problem is ____ $_{\text{twelve}}$, and thus 16

25 $_{\text{twelve}}$ − E $_{\text{twelve}}$ = ____ $_{\text{twelve}}$. 16

74.

$$\begin{array}{r} 83_{\text{twelve}} \\ -\text{T}_{\text{twelve}} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 85_{\text{twelve}} \\ -\_\_\_\__{\text{twelve}} \\ \hline \end{array}$$

10

Thus 83 $_{\text{twelve}}$ − T $_{\text{twelve}}$ = ____ $_{\text{twelve}}$. 75

75.

$$\begin{array}{r} 44_{\text{twelve}} \\ -9_{\text{twelve}} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 47_{\text{twelve}} \\ -\_\_\_\__{\text{twelve}} \\ \hline \end{array}$$

10

Thus 44 $_{\text{twelve}}$ − 9 $_{\text{twelve}}$ = ____ $_{\text{twelve}}$. 37

76. $\begin{array}{r} 62_{\text{twelve}} \\ -E_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 53

77. $\begin{array}{r} 62_{\text{twelve}} \\ -4_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 5T

78. $\begin{array}{r} 32_{\text{twelve}} \\ -18_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 16

79. $\begin{array}{r} 48_{\text{twelve}} \\ -2T_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 1T

80. $\begin{array}{r} E4_{\text{twelve}} \\ -9T_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 16

81. $\begin{array}{r} 4E3_{\text{twelve}} \\ -TT_{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} \end{array}$ 405

82. Sometimes we have to use the equal additions method twice:

$$\begin{array}{r} 100_{\text{twelve}} \\ -76_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \_\_\_\__{\text{twelve}} \\ -80_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} 146_{\text{twelve}} \\ -\_\_\_\__{\text{twelve}} \\ \hline \end{array}$$

106

100

The answer to the last problem is ____ $_{\text{twelve}}$. And thus 46

$100_{\text{twelve}} - 76_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 46

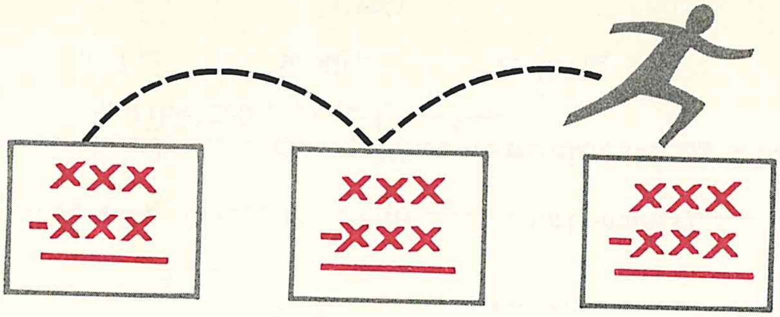
83.

$$\begin{array}{r} 306_{\text{twelve}} \\ -99_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \_\_\_\__{\text{twelve}} \\ -T0_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} 329_{\text{twelve}} \\ -\_\_\_\__{\text{twelve}} \\ \hline \end{array}$$

309

100

You can see that $306_{\text{twelve}} - 99_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 229



84.

$$\begin{array}{r} T32_{\text{twelve}} \\ -48E_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \_\_\_\_\_ _{\text{twelve}} \\ -490_{\text{twelve}} \\ \hline \end{array} \Rightarrow \begin{array}{r} \_\_\_\_\_ _{\text{twelve}} \\ -500_{\text{twelve}} \\ \hline \end{array}$$

T33, T63

Thus $T32_{\text{twelve}} - 48E_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$.

563

85. Use the equal additions method:

$$\begin{array}{r} E00_{\text{twelve}} \\ -98_{\text{twelve}} \\ \hline \\ \_\_\_\_\_ _{\text{twelve}} \end{array}$$

T24

86.

$$\begin{array}{r} 63T_{\text{twelve}} \\ -47E_{\text{twelve}} \\ \hline \\ \_\_\_\_\_ _{\text{twelve}} \end{array}$$

17E

87.

$$\begin{array}{r} 4073_{\text{twelve}} \\ -208E_{\text{twelve}} \\ \hline \\ \_\_\_\_\_ _{\text{twelve}} \end{array}$$

1ET4

88.

$$\begin{array}{r} 3421_{\text{twelve}} \\ -16TT_{\text{twelve}} \\ \hline \\ \_\_\_\_\_ _{\text{twelve}} \end{array}$$

1933

Multiplication in Base Twelve

89. Now let's take a look at multiplication. If we think of multiplication as a form of addition, we can say that

$2_{\text{twelve}} \times 5_{\text{twelve}} =$ ___ $_{\text{twelve}} +$ ___ $_{\text{twelve}}$. That is, 5, 5

$2_{\text{twelve}} \times 5_{\text{twelve}} =$ ___ $_{\text{twelve}}$. T

90. In base twelve,

$3_{\text{twelve}} \times 4_{\text{twelve}} =$ ___ $_{\text{twelve}} +$ ___ $_{\text{twelve}} +$ ___ $_{\text{twelve}}$. That is, 4, 4, 4

$3_{\text{twelve}} \times 4_{\text{twelve}} =$ ____ twelve(s) and ____ one(s). Thus 1, 0

$3_{\text{twelve}} \times 4_{\text{twelve}} =$ ___ $_{\text{twelve}}$. 10

91. $3_{\text{twelve}} \times 5_{\text{twelve}} =$ ____ twelve(s) and ____ one(s). In other 1, 3

words, $3_{\text{twelve}} \times 5_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 13

92. $4_{\text{twelve}} \times T_{\text{twelve}} =$ ____ twelve(s) and ____ one(s). That is, 3, 4

$4_{\text{twelve}} \times T_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 34

93. In base twelve, $4_{\text{twelve}} \times 9_{\text{twelve}} =$ —?—.

a. 36_{twelve} **b.** 30_{twelve} **c.** 40_{twelve} **b.** 30_{twelve}

94. And $5_{\text{twelve}} \times T_{\text{twelve}} =$ —?—.

a. 50_{twelve} **b.** $5T_{\text{twelve}}$ **c.** 42_{twelve} **c.** 42_{twelve}

95. $7_{\text{twelve}} \times 5_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 2E

96. $2_{\text{twelve}} \times E_{\text{twelve}} =$ ____ $_{\text{twelve}}$. 1T

97. Find the answer to each of these multiplication problems:

1. $6_{\text{twelve}} \times 5_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 26

2. $7_{\text{twelve}} \times 4_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 24

3. $8_{\text{twelve}} \times 8_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 54

4. $4_{\text{twelve}} \times 5_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 18

5. $9_{\text{twelve}} \times 8_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 60

98. Try these base twelve problems:

1. $3_{\text{twelve}} \times E_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 29

2. $5_{\text{twelve}} \times 2_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ T

3. $8_{\text{twelve}} \times T_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 68

4. $6_{\text{twelve}} \times E_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 56

5. $7_{\text{twelve}} \times T_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 5T

99. And, finally, try these:

1. $T_{\text{twelve}} \times 9_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 76

2. $T_{\text{twelve}} \times 5_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 42

3. $T_{\text{twelve}} \times T_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 84

4. $E_{\text{twelve}} \times E_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ T1

5. $E_{\text{twelve}} \times T_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$ 92.

100. Since you know that

$$3_{\text{twelve}} \times 5_{\text{twelve}} = 13_{\text{twelve}},$$

you can easily see that

$$30_{\text{twelve}} \times 5_{\text{twelve}} = \text{——?——}.$$

a. 130_{twelve} **b.** 150_{twelve} **c.** 170_{twelve}

a. 130_{twelve}

101. In base twelve, $4_{\text{twelve}} \times 40_{\text{twelve}} = \_\_\_\_\_\_\_\_ _{\text{twelve}}$. 140

102. And $5_{\text{twelve}} \times 20_{\text{twelve}} =$ ________ twelve. TO

103. Find the answer to each of these multiplication problems:

1. $7_{\text{twelve}} \times 20_{\text{twelve}} =$ ________ twelve 120
2. $6_{\text{twelve}} \times 30_{\text{twelve}} =$ ________ twelve 160
3. $5_{\text{twelve}} \times 50_{\text{twelve}} =$ ________ twelve 210
4. $7_{\text{twelve}} \times 50_{\text{twelve}} =$ ________ twelve 2E0
5. $9_{\text{twelve}} \times 60_{\text{twelve}} =$ ________ twelve 460

104. In base twelve, $7_{\text{twelve}} \times \text{T0}_{\text{twelve}} =$ ________ twelve. 5T0

105. $4_{\text{twelve}} \times \text{E0}_{\text{twelve}} =$ ________ twelve. 380

106. $\text{T}_{\text{twelve}} \times 70_{\text{twelve}} =$ ________ twelve. 5T0

107. $\text{E}_{\text{twelve}} \times \text{T0}_{\text{twelve}} =$ ________ twelve. 920

108. And now try these:

1. $\text{T}_{\text{twelve}} \times 50_{\text{twelve}} =$ ________ twelve 420
2. $5_{\text{twelve}} \times \text{E0}_{\text{twelve}} =$ ________ twelve 470
3. $2_{\text{twelve}} \times \text{E0}_{\text{twelve}} =$ ________ twelve 1T0
4. $3_{\text{twelve}} \times \text{T0}_{\text{twelve}} =$ ________ twelve 260
5. $\text{T}_{\text{twelve}} \times \text{T0}_{\text{twelve}} =$ ________ twelve 840

109. In base twelve, $30_{\text{twelve}} \times 20_{\text{twelve}} = $ —?—.

a. $5T0_{\text{twelve}}$ **b.** 600_{twelve} **c.** 6000_{twelve}

b. 600_{twelve}

110. $E0_{\text{twelve}} \times 20_{\text{twelve}} = $ —?—.

a. $1T00_{\text{twelve}}$ **b.** $2E00_{\text{twelve}}$ **c.** 2200_{twelve}

a. $1T00_{\text{twelve}}$

111. $E00_{\text{twelve}} \times 30_{\text{twelve}} = $ —?—.

a. $3E000_{\text{twelve}}$ **b.** 33000_{twelve} **c.** 29000_{twelve}

c. 29000_{twelve}

112. $T00_{\text{twelve}} \times 500_{\text{twelve}} = $ ________ $_{\text{twelve}}$.

420000

113. Try these multiplication problems in base twelve:

1. $E0_{\text{twelve}} \times T00_{\text{twelve}} = $ ________ $_{\text{twelve}}$ — 92000
2. $E00_{\text{twelve}} \times 600_{\text{twelve}} = $ ________ $_{\text{twelve}}$ — 560000
3. $40_{\text{twelve}} \times 300_{\text{twelve}} = $ ________ $_{\text{twelve}}$ — 10000
4. $50_{\text{twelve}} \times 5000_{\text{twelve}} = $ ________ $_{\text{twelve}}$ — 210000

114. In solving the problem $2_{\text{twelve}} \times 14_{\text{twelve}}$ you can write the two partial products:

$2_{\text{twelve}} \times 4_{\text{twelve}} = $ ______ $_{\text{twelve}}$ — 8

$2_{\text{twelve}} \times 10_{\text{twelve}} = $ ______ $_{\text{twelve}}$ — 20

The sum of the partial products is ______ $_{\text{twelve}}$, and thus — 28

$2_{\text{twelve}} \times 14_{\text{twelve}} = $ ______ $_{\text{twelve}}$. — 28

115. Let's try $3_{\text{twelve}} \times 16_{\text{twelve}}$:

$3_{\text{twelve}} \times 6_{\text{twelve}} = $ ______ $_{\text{twelve}}$ — 16

$3_{\text{twelve}} \times 10_{\text{twelve}} = $ ______ $_{\text{twelve}}$ — 30

Thus $3_{\text{twelve}} \times 16_{\text{twelve}} = $ ______ $_{\text{twelve}}$. — 46

116. To find the answer to $T_{twelve} \times 43_{twelve}$, you write:

$T_{twelve} \times 3_{twelve} =$ ______ $_{twelve}$ — 26

$T_{twelve} \times$ ______ $_{twelve} =$ ______ $_{twelve}$ — 40, 340

Thus $T_{twelve} \times 43_{twelve} =$ ______ $_{twelve}$. — 366

117. Now try $T_{twelve} \times 245_{twelve}$:

$T_{twelve} \times 5_{twelve} =$ ______ $_{twelve}$ — 42

$T_{twelve} \times$ ______ $_{twelve} =$ ______ $_{twelve}$ — 40, 340

$T_{twelve} \times$ ______ $_{twelve} =$ ______ $_{twelve}$ — 200, 1800

Adding the partial products, you find that

$T_{twelve} \times 245_{twelve} =$ ______ $_{twelve}$. — 1E82

118. Try $T_{twelve} \times 63_{twelve}$:

The answer is ______ $_{twelve}$. — 526

119. Try $T_{twelve} \times 162_{twelve}$:

The answer is ______ $_{twelve}$. — 1318

120. Try $E_{twelve} \times 1234_{twelve}$:

The answer is ______ $_{twelve}$. — 11108

121. When we multiply 23_{twelve} by 45_{twelve}, we have four partial products:

$3_{\text{twelve}} \times 5_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 13

$3_{\text{twelve}} \times 40_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 100

$20_{\text{twelve}} \times 5_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ T0

$20_{\text{twelve}} \times 40_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 800

When you find the sum of the partial products, you find that

$23_{\text{twelve}} \times 45_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$. 9E3

122.

$$\begin{array}{r} 68_{\text{twelve}} \\ \times 2T_{\text{twelve}} \\ \hline \end{array}$$

$T_{\text{twelve}} \times 8_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 68

$T_{\text{twelve}} \times 60_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 500

$20_{\text{twelve}} \times 8_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 140

$20_{\text{twelve}} \times 60_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 1000

Thus, $2T_{\text{twelve}} \times 68_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$. 16T8

123.

$$\begin{array}{r} 6T_{\text{twelve}} \\ \times E5_{\text{twelve}} \\ \hline \end{array}$$

$5_{\text{twelve}} \times T_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 42

$5_{\text{twelve}} \times 60_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 260

$E0_{\text{twelve}} \times T_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 920

$E0_{\text{twelve}} \times 60_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$ 5600

Thus, $E5_{\text{twelve}} \times 6T_{\text{twelve}} = \underline{\qquad\qquad}_{\text{twelve}}$. 6602

124. This one has six partial products:

$$\begin{array}{r} 3E4_{\text{twelve}} \\ \times TE_{\text{twelve}} \\ \hline \end{array}$$

$E_{\text{twelve}} \times 4_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 38

$E_{\text{twelve}} \times E0_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ T10

$E_{\text{twelve}} \times 300_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 2900

$T0_{\text{twelve}} \times 4_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 340

$T0_{\text{twelve}} \times E0_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 9200

$T0_{\text{twelve}} \times 300_{\text{twelve}} = \_\_\_\_\_\__{\text{twelve}}$ 26000

Thus $TE_{\text{twelve}} \times 3E4_{\text{twelve}} = \_\_\_\_\_\_\_\__{\text{twelve}}$. 37088

125. Try this one in base twelve:

$$\begin{array}{r} E3_{\text{twelve}} \\ \times E4_{\text{twelve}} \\ \hline \end{array}$$

$\_\_\_\_\_\__{\text{twelve}}$ T760

126. Now try this one:

$$\begin{array}{r} T9_{\text{twelve}} \\ \times 9T_{\text{twelve}} \\ \hline \end{array}$$

$\_\_\_\_\_\__{\text{twelve}}$ 8986

127. And finally, try this one:

$$\begin{array}{r} 121_{\text{twelve}} \\ \times 87_{\text{twelve}} \\ \hline \end{array}$$

$\_\_\_\_\_\__{\text{twelve}}$ T0T7

Division in Base Twelve

128. Just as we find partial products in multiplication problems, we find partial quotients in division problems. In base twelve the partial quotients will be 1_{twelve}, 10_{twelve}, 100_{twelve}, 1000_{twelve}, and so forth.

For example, $10_{\text{twelve}} \div 4_{\text{twelve}}$ looks like this:

$$\begin{array}{rl} 4_{\text{twelve}}\,\overline{)\,10_{\text{twelve}}} & \\ 4_{\text{twelve}} & \quad 1_{\text{twelve}} \\ \hline 8_{\text{twelve}} & \\ \underline{\_\_\_\__{\text{twelve}}} & \quad \_\_\_\__{\text{twelve}} \\ \_\_\_\__{\text{twelve}} & \\ 4_{\text{twelve}} & \quad \_\_\_\__{\text{twelve}} \\ \hline 0_{\text{twelve}} & \end{array}$$

4, 1

4

1

Adding the partial quotients, you find that

$10_{\text{twelve}} \div 4_{\text{twelve}} = \_\_\_\__{\text{twelve}}$. 3

129. Here's another one:

$$\begin{array}{rl} 45_{\text{twelve}}\,\overline{)\,925_{\text{twelve}}} & \\ \underline{\_\_\_\__{\text{twelve}}} & \quad 10_{\text{twelve}} \\ \_\_\_\__{\text{twelve}} & \\ 450_{\text{twelve}} & \quad \_\_\_\__{\text{twelve}} \\ \hline \_\_\_\__{\text{twelve}} & \\ 45_{\text{twelve}} & \quad \_\_\_\__{\text{twelve}} \\ \hline 0_{\text{twelve}} & \end{array}$$

450

495

10

45

1

Thus $925_{\text{twelve}} \div 45_{\text{twelve}} = \_\_\_\__{\text{twelve}}$. 21

130. $$\begin{array}{rl} ET_{\text{twelve}}\,\overline{)\,207T_{\text{twelve}}} & \\ \_\_\_\__{\text{twelve}} & \quad 10_{\text{twelve}} \\ \hline 109T_{\text{twelve}} & \\ \underline{\_\_\_\__{\text{twelve}}} & \quad \_\_\_\__{\text{twelve}} \\ \_\_\_\__{\text{twelve}} & \\ ET_{\text{twelve}} & \quad \_\_\_\__{\text{twelve}} \\ \hline 0_{\text{twelve}} & \end{array}$$

ET0

ET0, 10

ET

1

Thus $207T_{\text{twelve}} \div ET_{\text{twelve}} = \_\_\_\__{\text{twelve}}$. 21

131. $9TE_{\text{twelve}}\overline{)E768E_{\text{twelve}}}$

$9TE00_{\text{twelve}}$ ______ twelve | 100

$1878E_{\text{twelve}}$

______ twelve ______ twelve | 9TE0, 10

$T89E_{\text{twelve}}$

$9TE0_{\text{twelve}}$ ______ twelve | 10

$9TE_{\text{twelve}}$

$9TE_{\text{twelve}}$ ______ twelve | 1

0_{twelve}

And so $E768E_{\text{twelve}} \div 9TE_{\text{twelve}} =$ ______ twelve. | 121

132. $T8_{\text{twelve}}\overline{)1T28_{\text{twelve}}}$

______ twelve ______ twelve | T80, 10

______ twelve | E68

______ twelve ______ twelve | T80, 10

______ twelve | T8

______ twelve ______ twelve | T8, 1

0_{twelve}

Thus $1T28_{\text{twelve}} \div T8_{\text{twelve}} =$ ______ twelve. | 21

133. Try this one on scrap paper:

$E6_{\text{twelve}}\overline{)21T6_{\text{twelve}}}$

The answer is ______ twelve. | 23

134. Now try this one:

$TE4_{\text{twelve}}\overline{)2T7T8_{\text{twelve}}}$

The answer is ______ twelve. | 32

135. And finally, try this one:

$4T_{\text{twelve}}\overline{)4T98_{\text{twelve}}}$

The answer is ______ twelve. | 102

Try Your Hand

■ The answers to these questions appear on page 171.

1. Transform the following base twelve numerals to base ten:

   a. 14_{twelve}
   b. $1E_{\text{twelve}}$
   c. 33_{twelve}
   d. 56_{twelve}
   e. $T0_{\text{twelve}}$
   f. TE_{twelve}
   g. 104_{twelve}
   h. 163_{twelve}
   i. $1T0_{\text{twelve}}$
   j. $20E_{\text{twelve}}$

2. Now transform these base ten numerals to base twelve:

   a. 27_{ten}
   b. 39_{ten}
   c. 46_{ten}
   d. 71_{ten}
   e. 84_{ten}
   f. 87_{ten}
   g. 100_{ten}
   h. 106_{ten}
   i. 130_{ten}
   j. 143_{ten}

3. Find answers to these addition problems:

   a. $16_{\text{twelve}} + 8_{\text{twelve}}$
   b. $1T_{\text{twelve}} + 6_{\text{twelve}}$
   c. $T0_{\text{twelve}} + E_{\text{twelve}}$
   d. $35_{\text{twelve}} + 2E_{\text{twelve}}$
   e. $ET_{\text{twelve}} + 3T_{\text{twelve}}$

4. Find an answer to each of these base twelve subtraction problems:

   a. $1T_{\text{twelve}} - 7_{\text{twelve}}$
   b. $40_{\text{twelve}} - E_{\text{twelve}}$
   c. $6E_{\text{twelve}} - 40_{\text{twelve}}$
   d. $73_{\text{twelve}} - 3T_{\text{twelve}}$
   e. $100_{\text{twelve}} - 5E_{\text{twelve}}$

5. Here are some multiplication problems in base twelve. Find the answer to each:

   a. $15_{\text{twelve}} \times 7_{\text{twelve}}$
   b. $42_{\text{twelve}} \times T_{\text{twelve}}$
   c. $16_{\text{twelve}} \times 12_{\text{twelve}}$
   d. $34_{\text{twelve}} \times 1E_{\text{twelve}}$
   e. $4T_{\text{twelve}} \times 11_{\text{twelve}}$

6. Find answers to these base twelve division problems:

   a. $10_{\text{twelve}} \div 4_{\text{twelve}}$
   b. $2E_{\text{twelve}} \div 7_{\text{twelve}}$
   c. $1ET_{\text{twelve}} \div 1T_{\text{twelve}}$
   d. $286_{\text{twelve}} \div 13_{\text{twelve}}$
   e. $925_{\text{twelve}} \div 45_{\text{twelve}}$

THINK IT OVER

■ Try these questions. They will cause you to think hard about base twelve. In some cases, you will be forced to extend what you learned in this chapter.

1. The Duodecimal Society of America has long tried to introduce the use of base twelve numerals in everyday affairs. What advantages

does base twelve have over base ten? What disadvantages might there be in making a change from base ten to base twelve?

2. Why are the symbols T and E used in base twelve? *Must* we use these symbols?

3. How can you use your knowledge of base twelve to help you solve problems involving measurement of time, distance, and weight? Where else in the world about you do base twelve ideas play an important role?

SUMMING UP

■ You can use the following questions to help you review what you have learned in this program. Try each question and then turn to page 171 to see whether you answered correctly.

1. Transform the following numerals to base ten:

   a. 13_{five}
   b. 33_{four}
   c. 47_{nine}
   d. 100_{four}
   e. 211_{five}

2. Transform the following base ten numerals to the base indicated.

   a. $3_{\text{ten}} = ?_{\text{five}}$
   b. $8_{\text{ten}} = ?_{\text{four}}$
   c. $15_{\text{ten}} = ?_{\text{seven}}$
   d. $26_{\text{ten}} = ?_{\text{five}}$
   e. $23_{\text{ten}} = ?_{\text{three}}$

3. Find answers to the following addition problems:

   a. $21_{\text{eight}} + 3_{\text{eight}}$
   b. $16_{\text{nine}} + 15_{\text{nine}}$
   c. $22_{\text{three}} + 12_{\text{three}}$
   d. $414_{\text{six}} + 305_{\text{six}}$
   e. $44_{\text{five}} + 30_{\text{five}}$

4. Find answers to these subtraction problems:

   a. $46_{\text{nine}} - 5_{\text{nine}}$
   b. $21_{\text{five}} - 4_{\text{five}}$
   c. $50_{\text{seven}} - 31_{\text{seven}}$
   d. $102_{\text{six}} - 20_{\text{six}}$
   e. $221_{\text{four}} - 132_{\text{four}}$

5. Try these multiplication problems:

   a. $4_{\text{five}} \times 40_{\text{five}}$
   b. $30_{\text{four}} \times 110_{\text{four}}$
   c. $26_{\text{eight}} \times 30_{\text{eight}}$
   d. $44_{\text{six}} \times 22_{\text{six}}$
   e. $22_{\text{three}} \times 120_{\text{three}}$

6. Find answers to the following division problems:

a. $410_{\text{six}} \div 10_{\text{six}}$
b. $26_{\text{nine}} \div 8_{\text{nine}}$
c. $211_{\text{five}} \div 24_{\text{five}}$
d. $376_{\text{nine}} \div 28_{\text{nine}}$
e. $511_{\text{seven}} \div 32_{\text{seven}}$

7. Find the missing part in each example below:

a. $3_{\text{five}} + ?_{\text{five}} = 12_{\text{five}}$
b. $12_{\text{three}} - ?_{\text{three}} = 1_{\text{three}}$
c. $?_{\text{eight}} \div 2_{\text{eight}} = 4_{\text{eight}}$
d. $6_{\text{seven}} \times ?_{\text{seven}} = 42_{\text{seven}}$
e. $42_{\text{five}} + ?_{\text{five}} = 103_{\text{five}}$

8. Transform each of the following numerals to base ten:

a. 10_{two}
b. 10_{twelve}
c. 100_{two}
d. $3T_{\text{twelve}}$
e. 110_{two}

9. Transform each of these base ten numerals to the base indicated:

a. $4_{\text{ten}} = ?_{\text{two}}$
b. $9_{\text{ten}} = ?_{\text{two}}$
c. $11_{\text{ten}} = ?_{\text{twelve}}$
d. $15_{\text{ten}} = ?_{\text{twelve}}$
e. $24_{\text{ten}} = ?_{\text{twelve}}$

10. Find answers to the following examples:

a. $T_{\text{twelve}} + 5_{\text{twelve}}$
b. $15_{\text{twelve}} - 7_{\text{twelve}}$
c. $100_{\text{two}} - 1_{\text{two}}$
d. $10101_{\text{two}} \div 111_{\text{two}}$
e. $110_{\text{two}} \times 11_{\text{two}}$

Do It Yourself

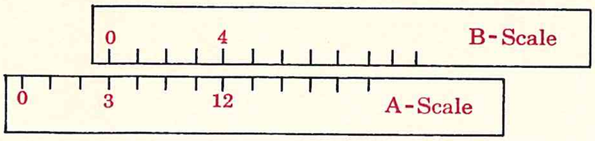
Here are several projects which will help you to learn more about number bases. Do these and challenge yourself to think about other projects you could attempt.

1. You can make a "base wheel" for transforming numerals from one base to another. Cut out a circular piece of cardboard and draw on it several concentric (having the same center) circles. In one ring, write the base ten numerals from 1 to 20. In another ring, write the corresponding base nine numerals. (Be sure to line up each base nine numeral with the proper base ten numeral.) Then use another ring for base eight. You can continue until you have used up all the rings. You'll have a very easy job transforming numerals from one base to another.

2. It is possible to make a *slide rule* for addition in various number bases. Here's how to make such a slide rule for base five: First take two strips of cardboard, label them *A* and *B*, and mark off twenty or twenty-five equally spaced points on one edge of each strip.

Then label the points consecutively, using base five numerals.

To add 4_{five} to 3_{five}, we place the 0 mark of the *B*-scale next to the 3 mark of the *A*-scale.



Then we locate the 4 mark on the *B*-scale. It matches up with 12_{five} on the *A*-scale. Thus, $4_{\text{five}} + 3_{\text{five}} = 12_{\text{five}}$.

Make a slide rule for base nine. How can you use this slide rule for *addition?* How can you use it for *subtraction?* Try to invent a similar device for *multiplication* in base nine.

3. What does a fraction such as $\frac{1}{2}$ mean if the numerals are written in base eight? If in base ten, the decimal equivalent of $\frac{1}{2}$ is .5, what is the *octal* (base eight) equivalent of $\frac{1}{2}$? Find ways to express the octal equivalents of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$. Prepare a report showing relationships and differences between octal and decimal representations of fractions.

4. Do some research and write a paper on the use of base two in computer work.

5. Do some research to learn about arguments in favor of replacing the decimal (base ten) system with the duodecimal (base twelve) system.

Looking Ahead

If you want to extend your knowledge of number bases, and if you want to do some very interesting reading of mathematics in general, you should take a look at the following books.

Andrews, F. Emerson. *New Numbers.* Harcourt, Brace & World, 1935.

Bakst, Aaron. *Mathematics, Its Magic and Mastery.* D. Van Nostrand Co., 1952.

Johnson, Dovovan A., and Glenn, William H. *Understanding Numeration Systems.* Webster Publishing Co., 1960.

School Mathematics Study Group. *Mathematics for the Junior High School (Revised Edition), Volume I, Part I.* Yale University Press, 1961.

Terry, George S. *The Dozen System.* Longmans, Green and Co., 1941.

TRY YOUR HAND ANSWERS FOR CHAPTER 1

1. a. 5_{ten} b. 7_{ten} c. 36_{ten} d. 2_{ten} e. 8_{ten} f. 11_{ten} g. 36_{ten} h. 15_{ten} i. 63_{ten} j. 30_{ten}

2. a. 210_{three} b. 111_{four} c. 41_{five} d. 33_{six} e. 30_{seven} f. 25_{eight} g. 23_{nine} h. 19_{twelve} i. 16_{fifteen} j. $10_{\text{twenty-one}}$

3. a. 20_{five} b. 20_{six} c. 3_{seven} d. 11_{nine} e. 13_{four} f. 110_{three} g. 20_{twelve} h. 18_{twenty} i. 1_{thirty} j. 10_{fifty}

4. a. base eight b. base six c. base seven d. base eight e. base six f. base three g. base seven h. base five i. base eight j. base thirteen

5. a. 1_{ten} b. 2_{ten} c. 7_{ten} d. 3_{ten} e. 4_{ten} f. 2_{ten} g. 6_{ten} h. 5_{ten} i. 8_{ten} j. 6_{ten}

6. a. base six b. base three c. base seven d. base eight e. base five f. base four g. base three h. base thirty i. base two j. base twelve

7. a. 3_{ten} b. 4_{ten} c. 5_{ten} d. 6_{ten} e. 7_{ten} f. 8_{ten} g. 9_{ten} h. 10_{ten} i. 16_{ten} j. 64_{ten}

8. a. 75_{ten} b. 61_{ten} c. 111_{ten} d. 160_{ten} e. 400_{ten} f. 420_{ten} g. 424_{ten} h. 800_{ten} i. 1226_{ten} j. 625_{ten}

9. a. 1_{five}, 3_{five}, 10_{five}, 12_{five}, 14_{five}, 21_{five}
   b. 53_{six}, 55_{six}, 101_{six}, 103_{six}, 105_{six}
   c. 17_{nine}, 15_{nine}, 13_{nine}, 11_{nine}, 8_{nine}, 6_{nine}
   d. 1_{three}, 10_{three}, 12_{three}, 21_{three}, 100_{three}, 102_{three}
   e. 1_{eight}, 5_{eight}, 11_{eight}, 15_{eight}, 21_{eight}, 25_{eight}, 31_{eight}, 35_{eight}
   f. 100_{four}, 30_{four}, 20_{four}, 10_{four}, 0_{four}
   g. 33_{ten}, 40_{ten}, 47_{ten}, 54_{ten}, 61_{ten}
   h. 5_{six}, 12_{six}, 15_{six}, 22_{six}, 25_{six}, 32_{six}
   i. 21_{ten}, 26_{ten}, 31_{ten}, 36_{ten}, 41_{ten}, 46_{ten}, 51_{ten}, 56_{ten}
   j. 2_{three}, 12_{three}, 22_{three}, 102_{three}, 112_{three}, 122_{three}, 202_{three}

TRY YOUR HAND ANSWERS FOR CHAPTER 2

1. a. 1030_{five} b. 1362_{eight} c. 1003_{six} d. 133_{six} e. 121_{three} f. 233_{four} g. 46_{nine} h. 170_{eight} i. 2010_{three} j. 2012_{five}

2. a. 1210_{three} b. 121_{three} c. 220_{three} d. 2001_{three}

3. a. 24_{ten} b. 127_{ten} c. 211_{ten} d. 405_{ten} e. 63_{ten} f. 300_{ten} g. 3300_{ten} h. 235_{ten} i. 621_{ten} j. 410_{ten}

4. a. base seven b. base eight c. base five d. base four e. base seven f. base four g. base eleven h. base fourteen i. base two j. base seventeen

5. a. 4
   b. 7
   c. 5
   d. 11
   e. 15
   f. 11
   g. 12
   h. 6
   i. 8
   j. 10

6. a. 12_{seven}
   b. 7_{eight}
   c. 12_{seven}
   d. 12_{seven}
   e. 15_{nine}
   f. 28_{nine}
   g. 27_{eight}
   h. 41_{six}
   i. 42_{ten}
   j. 113_{seven}

7. a. Yes
   b. No
   c. Yes
   d. Yes
   e. No

8. a. 7_{eleven}
   b. 66_{eleven}
   c. 10_{eleven}
   d. 22_{eleven}
   e. 1800_{eleven}
   f. 3300_{eleven}
   g. 1347_{eleven}
   h. 223_{eleven}
   i. 140_{eleven}
   j. 124_{eleven}

9. a. 9_{eleven}
   b. 17_{twelve}
   c. 20_{sixteen}
   d. 50_{fifteen}
   e. 21_{twelve}
   f. 68_{twenty}
   g. 51_{fifteen}
   h. 72_{twelve}
   i. 520_{fifteen}
   j. 50_{fifty}

TRY YOUR HAND ANSWERS FOR CHAPTER 3

1. a. 3_{nine}
   b. 12_{four}
   c. 113_{eight}
   d. 105_{seven}
   e. 3200_{six}
   f. 1002_{five}
   g. 3653_{ten}
   h. 1025_{nine}
   i. 3000_{four}
   j. 1645_{eight}

2. a. 2_{three}
   b. 3_{four}
   c. 4_{five}
   d. 5_{six}
   e. 6_{seven}
   f. 7_{eight}
   g. 8_{nine}
   h. 9_{ten}

3. a. 0_{nine}
   b. 1_{nine}
   c. 2_{nine}
   d. 3_{nine}
   e. 4_{nine}
   f. 5_{nine}
   g. 6_{nine}
   h. 7_{nine}
   i. 8_{nine}
   j. 10_{nine}

4. a. 1_{five}
   b. 10_{five}
   c. 100_{five}
   d. 1_{five}
   e. 10_{five}
   f. 11_{five}
   g. 111_{five}
   h. 320_{five}
   i. 20_{five}
   j. 312_{five}

5. a. 13_{ten}
   b. 24_{seven}
   c. 54_{nine}
   d. 80_{nine}
   e. 134_{six}
   f. 124_{eight}
   g. 88_{ten}
   h. 6_{seven}
   i. 21_{four}
   j. 444_{five}

6. a. 8_{ten}
   b. 24_{ten}
   c. 34_{ten}
   d. 39_{ten}
   e. 53_{ten}
   f. 97_{ten}
   g. 78_{ten}
   h. 139_{ten}
   i. 298_{ten}
   j. 485_{ten}

7. a. 51_{six}
   b. 303_{four}
   c. 342_{five}
   d. 7368_{nine}
   e. 2012_{three}

8. a. 4
   b. 31
   c. 10
   d. 30
   e. 8
   f. 43
   g. 61
   h. 30
   i. 7

9. a. base ten
   b. base seven
   c. base nine
   d. base eight
   e. base three

10. a. 20_{twenty}
    b. 4_{twelve}
    c. 27_{sixteen}
    d. 27_{twelve}
    e. 7_{fourteen}

11. a. 7_{ten}
    b. 4_{five}
    c. 15_{seven}
    d. 22_{three}
    e. 115_{six}

TRY YOUR HAND ANSWERS FOR CHAPTER 4

1. a. 41_{five}
   b. 40_{five}
   c. 220_{five}
   d. 1120_{five}
   e. 211_{five}

2. a. 40_{six}
   b. 175_{eight}
   c. 121_{three}
   d. 162_{ten}
   e. 330_{five}
   f. 14300_{six}
   g. 563_{nine}
   h. 3015_{seven}
   i. 1425_{six}
   j. 1470_{ten}

3. a. 104_{ten}
   b. 161_{ten}
   c. 963_{ten}
   d. 320_{ten}
   e. 1320_{ten}
   f. 369_{ten}
   g. 462_{ten}
   h. 312_{ten}
   i. 1750_{ten}
   j. 374_{ten}

4. a. 6534_{ten}
   b. 408_{ten}
   c. 693_{ten}
   d. 665_{ten}
   e. 1034_{ten}
   f. 957_{ten}
   g. 1026_{ten}
   h. 46906_{ten}
   i. 3136_{ten}
   j. 879824_{ten}

5. a. 13_{six}
   b. 12_{seven}
   c. 11_{eight}
   d. 10_{nine}
   e. 9_{ten}
   f. 14_{six}
   g. 13_{seven}
   h. 12_{eight}
   i. 11_{nine}
   j. 10_{ten}

6. a. base eight
   b. base nine
   c. base five
   d. base four
   e. base seven

7. a. impossible
   b. base eight
   c. base six
   d. impossible
   e. base five
   f. impossible
   g. impossible
   h. base sixteen
   i. base twenty-one
   j. base thirteen

8. a. 4
   b. 7
   c. 2
   d. 5
   e. 30
   f. 2
   g. 4
   h. 15
   i. 3
   j. 23

9. a. 16_{nine}
   b. 17_{eight}
   c. 25_{eight}
   d. 46_{nine}
   e. 24_{eight}

10. a. 208_{ten}
    b. 216_{ten}
    c. 560_{ten}
    d. 176_{ten}
    e. 945_{ten}

TRY YOUR HAND ANSWERS FOR CHAPTER 5

1. a. 11_{eight}
   b. 4_{six}
   c. 7_{nine}
   d. 13_{six}
   e. 25_{nine}
   f. 16_{seven}
   g. 13_{four}
   h. 22_{nine}
   i. 21_{five}
   j. 11_{four}

2. a. 12_{five}
   b. 4_{seven}
   c. 36_{eight}
   d. 15_{ten}
   e. 22_{three}
   f. 43_{eight}
   g. 11_{seven}
   h. 221_{three}
   i. 130_{ten}
   j. 2_{four}

3. a. 13_{ten}
   b. 13_{seven}
   c. 13_{nine}
   d. 104_{eight}
   e. 18_{ten}
   f. 16_{nine}
   g. 102_{five}
   h. 102_{six}
   i. 11_{ten}
   j. 12_{ten}

4. a. Yes
   b. No
   c. No
   d. No
   e. No
   f. No
   g. Yes
   h. Yes
   i. Yes
   j. No

5. a. base five
   b. base nine
   c. base ten
   d. base six
   e. base three

6. a. No
   b. No
   c. Yes
   d. No
   e. No
   f. No
   g. No
   h. No
   i. Yes
   j. Yes

TRY YOUR HAND ANSWERS FOR CHAPTER 6

1. a. 3_{ten} b. 5_{ten} c. 7_{ten} d. 8_{ten} e. 10_{ten} f. 11_{ten} g. 15_{ten} h. 16_{ten} i. 18_{ten} j. 31_{ten}

2. a. 100_{two} b. 111_{two} c. 1100_{two} d. 1110_{two} e. 10010_{two} f. 11001_{two} g. 100000_{two} h. 100011_{two} i. 1000001_{two} j. 1000110_{two}

3. a. 4_{ten} b. 6_{ten} c. 11_{ten} d. 13_{ten} e. 16_{ten} f. 29_{ten} g. 21_{ten} h. 24_{ten} i. 30_{ten} j. 35_{ten}

4. a. 10_{two}, 100_{two}, 110_{two}
b. 1_{two}, 100_{two}, 111_{two}
c. 10_{two}, 110_{two}, 1010_{two}, 1110_{two}
d. 1000_{two}, 110_{two}, 100_{two}, 10_{two}
e. 1011_{two}, 111_{two}, 11_{two}

5. a. 111_{two} b. 1000_{two} c. 1010_{two} d. 10001_{two} e. 10100_{two}

6. a. 10_{two} b. 11_{two} c. 1_{two} d. 11_{two} e. 1011_{two}

7. a. 1001_{two} b. 1111_{two} c. 11001_{two} d. 101010_{two} e. 110001_{two}

8. a. 11_{two} b. 101_{two} c. 110_{two} d. 101_{two} e. 11_{two}

TRY YOUR HAND ANSWERS FOR CHAPTER 7

1. a. 16_{ten} b. 23_{ten} c. 39_{ten} d. 66_{ten} e. 120_{ten} f. 131_{ten} g. 148_{ten} h. 219_{ten} i. 264_{ten} j. 299_{ten}

2. a. 23_{twelve} b. 33_{twelve} c. $3T_{\text{twelve}}$ d. $5E_{\text{twelve}}$ e. 70_{twelve} f. 73_{twelve} g. 84_{twelve} h. $8T_{\text{twelve}}$ i. TT_{twelve} j. EE_{twelve}

3. a. 22_{twelve} b. 24_{twelve} c. TE_{twelve} d. 64_{twelve} e. 138_{twelve}

4. a. 13_{twelve} b. 31_{twelve} c. $2E_{\text{twelve}}$ d. 35_{twelve} e. 61_{twelve}

5. a. $9E_{\text{twelve}}$ b. 358_{twelve} c. 190_{twelve} d. 648_{twelve} e. $52T_{\text{twelve}}$

6. a. 3_{twelve} b. 5_{twelve} c. 11_{twelve} d. 22_{twelve} e. 21_{twelve}

ANSWERS TO SUMMING UP

1. a. 8_{ten} b. 15_{ten} c. 43_{ten} d. 16_{ten} e. 56_{ten}

2. a. 3 b. 20 c. 21 d. 101 e. 212

3. a. 24_{eight} b. 32_{nine} c. 111_{three} d. 1123_{six} e. 124_{five}

4. a. 41_{nine} b. 12_{five} c. 16_{seven} d. 42_{six} e. 23_{four}

5. a. 310_{five} b. 3300_{four} c. 1020_{eight} d. 1452_{six} e. 11110_{three}

6. a. 41_{six} b. 3_{nine} c. 4_{five} d. 13_{nine} e. 14_{seven}

7. a. 4_{five} b. 11_{three} c. 10_{eight} d. 5_{seven} e. 11_{five}

8. a. 2_{ten} b. 12_{ten} c. 4_{ten} d. 46_{ten} e. 6_{ten}

9. a. 100 b. 1001 c. E d. 13 e. 20

10. a. 13_{twelve} b. T_{twelve} c. 11_{two} d. 11_{two} e. 10010_{two}

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